

# Abstract Machines

*Summer Semester 2004*

*8. Homework*

*Deadline: 23 June 2004 12:00*

Exercise 1:

*8 Points*

All positive numbers can be declared as successors of 0, like:

$0 \equiv 0$   
 $1 \equiv \text{succ}(0)$   
 $2 \equiv \text{succ}(\text{succ}(0))$   
 $3 \equiv \text{succ}(\text{succ}(\text{succ}(0)))$   
 $\dots$

Write a prolog program including the following predicates:

- a) *less/2* where the first parameter is smaller than the second parameter. (e.g. *less(succ(0), succ(succ(succ(0))))*)
- b) *add/3* where the third parameter is the sum of the first two parameters. (e.g. *add(succ(0), succ(succ(0)), succ(succ(succ(0))))*)
- c) *mul/3* where the third parameter is the multiplication of the first two parameters.
- d) *fact/2* where second parameter is the factorial of the first parameter.

Exercise 2:

*6 Points*

We represent trees using the terms *leaf* and *node(s, t, u)* where *s* is some value, and *t* and *u* are trees. Write a prolog program including the following predicates:

- a) *size/2* where the second parameter is the size of the first parameter which is a tree. (e.g. *size(node(a, leaf, node(b, leaf, leaf)), succ(succ(0))))*)
- b) *member/2* where the first parameter occurs at some node in the second parameter which is a tree. (e.g. *member(a, node(a, leaf, leaf)))*)
- c) *insert/3* where third parameter is obtained by inserting the first parameter into the second parameter which is a tree. (e.g. *insert(a, node(b, empty, empty), node(b, node(a, empty, empty), empty)))*)

Exercise 3:

*6 Points*

Unify *t* and *s*

$$\begin{aligned} t &\equiv p(X_1, \dots, X_n) \\ s &\equiv p(f(X_0, X_0), f(X_1, X_1), \dots, f(X_{n-1}, X_{n-1})) \end{aligned}$$

Compare the costs for the unification with and without occur check!