Cryptography and cryptographic protocols

- Cryptography deals with algorithms for encryption, decryption, random number generation, etc. Cryptographic protocols use cryptography for exchanging messages.
- Attacks against cryptographic primitives involves breaking the algorithm for encryption, etc. Attacks against cryptographic protocols may be of completely logical nature.
- Cryptographic protocols may be insecure even if the underlying cryptographic primitives are completely secure.
- Hence we often separate the study of cryptographic protocols from that of cryptographic primitives.

Difficulty in ensuring correctness of cryptographic protocols

- Infinitely many sessions
- Infinitely many participants
- Infinitely many nonces
- Sessions are interleaved
- Adversary can replace messages by any arbitrary message: infinitely branching system

Back to our example

1. $A \longrightarrow B : \{A, N_a\}_{K_b}$
2. $B \longrightarrow A : \{N_a, N_b\}_{K_a}$
3. $A \longrightarrow B : \{N_b\}_{K_b}$

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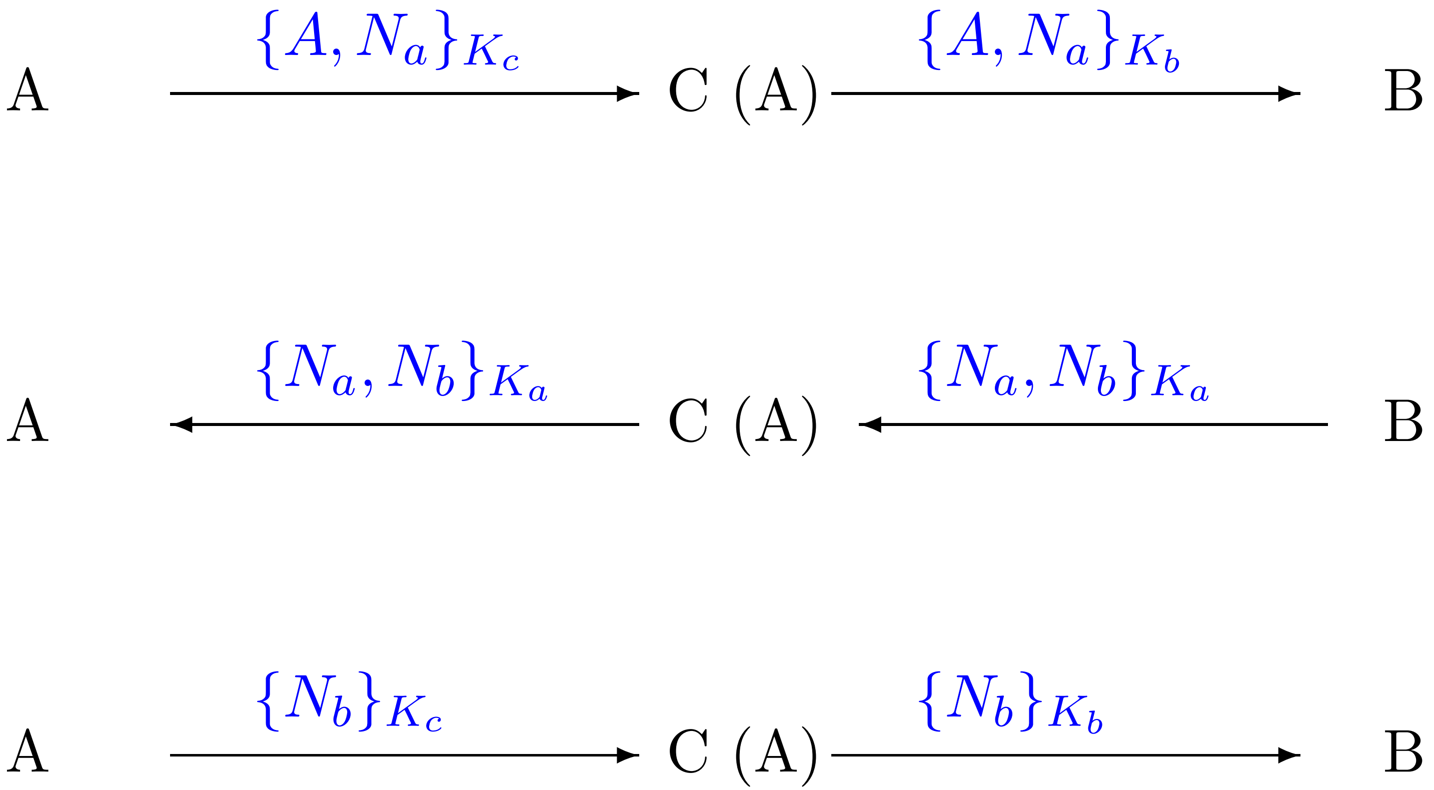
This is the well-known Needham-Schroeder public-key protocol.

Published in 1978. Attack found after 17 years in 1995 by Lowe.

Man in the middle attack



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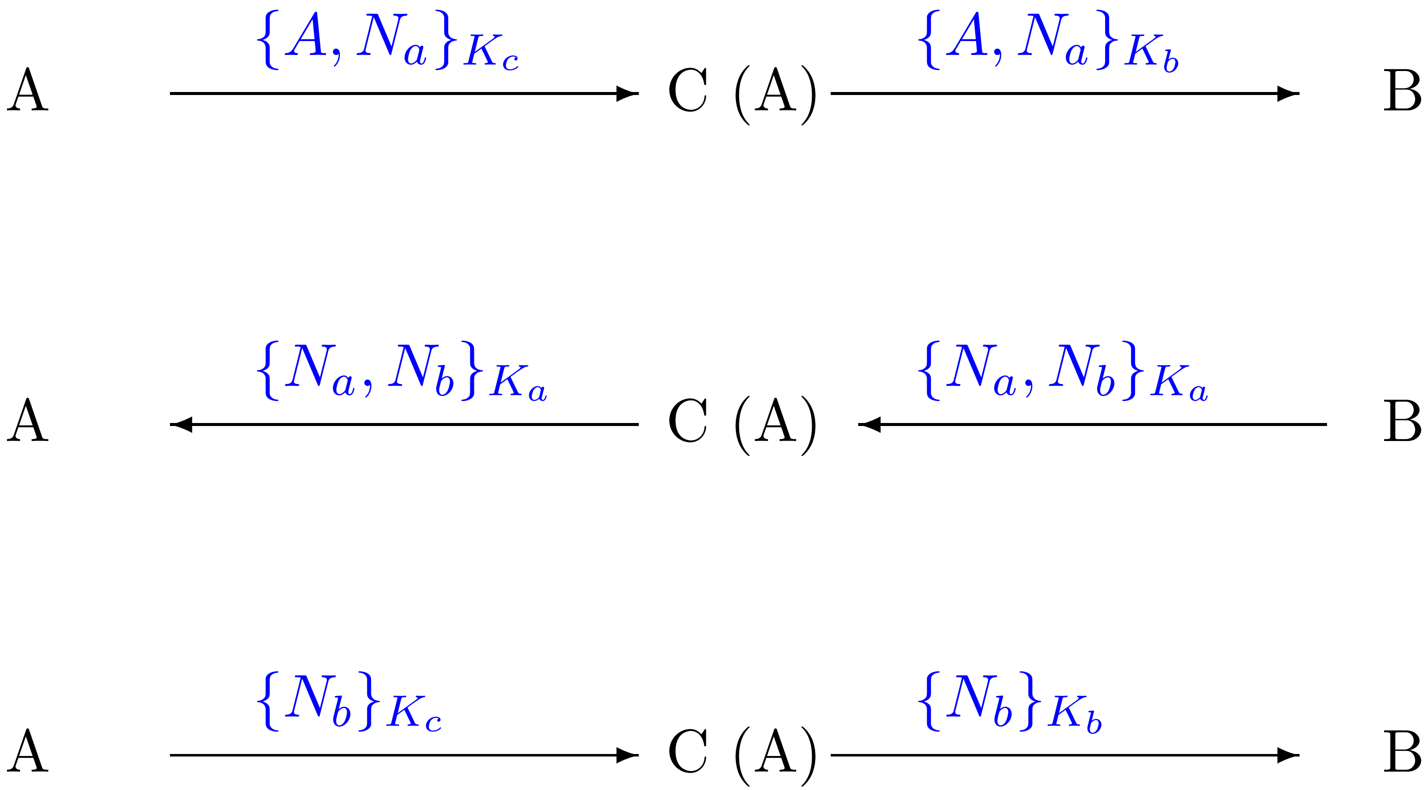

Even very simple protocols may have subtle flaws

Consequences

Suppose B is the server of a bank.

C, who can now pretend to be A:

$$C \longrightarrow B : \{N_a, N_b, \text{ transfer £5000 from account of } A \text{ to account of } C\}_{K_b}$$

A fix: the Needham-Schroeder-Lowe protocol [Lowe,1985]

B includes his identity in the message he sends:

1. $A \longrightarrow B : \{A, Na\}_{K_b}$
2. $B \longrightarrow A : \{B, N_a, N_b\}_{K_a}$
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Is it secure?

A variant of the Needham-Schroeder-Lowe protocol

Suppose now we change the place of B in the second message:

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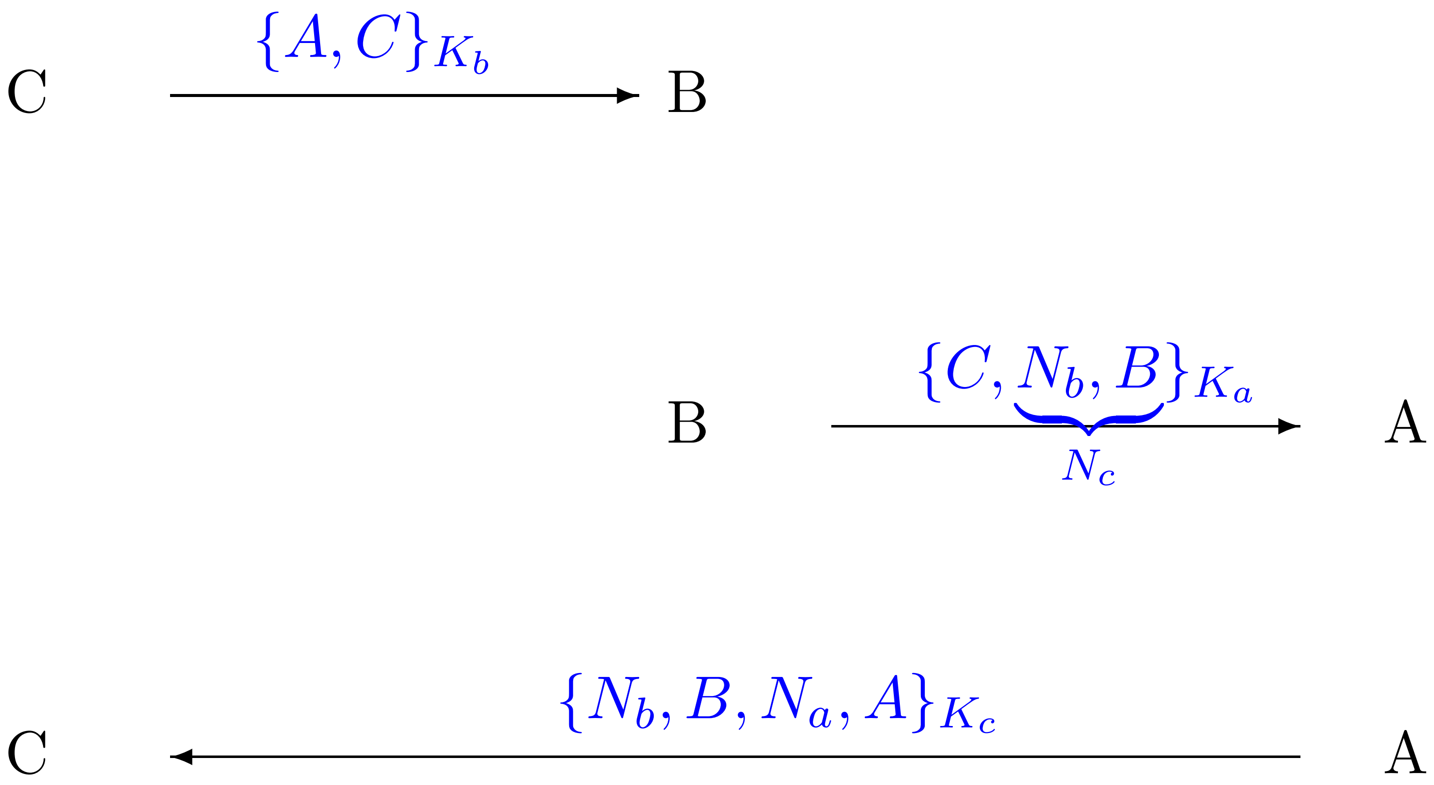
Does this affect security?

Type flaw

An attack on the variant of the Needham-Schroeder-Lowe protocol [Millen]:



The Spi calculus

Abadi, Gordon, 1997

- Extends pi calculus which provides a language for describing processes.
- We treat protocols as processes, where messages sent and received by processes may involve encryption.
- Security is defined as equivalence between processes in the eyes of an arbitrary environment.
- Environment is also a spi calculus process.
- We study information flow to check whether secrets are leaked.

- A process may involve sequences of actions for sending and receiving messages on channels.
- A Processes may contain smaller processes running in parallel.

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Use halt to denote a finished process: it does nothing.

We write $\mathsf{send}_c\langle M\rangle; P$ to denote a process that sends the message M on channel c after which it executes the process P.

$\mathsf{recv}_c(x); Q$ denotes a process that is listening on the channel c.

On receiving some message M on this channel then it executes process $Q[M/x]$.

The process

$$P_1 \triangleq \mathsf{recv}_c(x); \mathsf{send}_d\langle x\rangle; \mathsf{halt}$$

on receiving message M on channel c, sends M on channel d and then halts.

The process

$$P_2 \triangleq \mathsf{send}_c\langle M\rangle; \mathsf{halt}$$

sends M on channel c and halts.

The process

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The process

$$P_2 \triangleq \mathsf{send}_c\langle M \rangle; \mathsf{halt}$$

sends M on channel c and halts.

Putting them in parallel gives the process

$$P_3 \triangleq P_1 \mid P_2$$

The message sent by P_1 is received by P_2. Hence P_3 as a whole can make a "silent" transition to the process $\mathsf{send}_d\langle M \rangle; \mathsf{halt}$.

Further the process

$$P_5 \triangleq P_3 \mid P_4$$

where

$$P_4 \triangleq \mathsf{recv}_d(x); \mathsf{halt}$$

can halt after making only silent transitions.

Intuitively P_5 represents the protocol

$$P_2 \longrightarrow P_1 : \quad M \quad \text{(on channel } c\text{)}$$

$$P_1 \longrightarrow P_4 : \quad M \quad \text{(on channel } d\text{)}$$

We can restrict access to channels.

The process $\mathsf{new}\ c; P$ creates a fresh channel c and can be used inside process P. No outside process can access c.

(c is like a bound variable whose scope is inside P)

We consider processes to be the same after renaming of bound names.

Consider the process

$$(\mathsf{new}\ c; \mathsf{send}_c\langle M\rangle; \mathsf{halt}) \mid (\mathsf{recv}_c(x); \mathsf{halt})$$

No communication happens between the two smaller processes.

The above process is the same as the following one.

$$(\mathsf{new}\ d; \mathsf{send}_d\langle M\rangle; \mathsf{halt}) \mid (\mathsf{recv}_c(x); \mathsf{halt})$$

Hence new allows us to create channels for secure communication.

Consider the process

$$\mathsf{new}\ c; (\mathsf{send}_c\langle M\rangle; \mathsf{halt} \mid \mathsf{recv}_c(x); P \mid \mathsf{recv}_c(x); Q)$$

Communication can take place between first and second subprocess to create the process $\mathsf{new}\ c; (P[M/x] \mid \mathsf{recv}_c(x); Q)$

Or communication can take place between first and third subprocess to create the process $\mathsf{new}\ c; (\mathsf{recv}_c(x); P \mid Q[M/x])$

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However the process

$$(\mathsf{new}\ c; (\mathsf{send}_c\langle M\rangle; \mathsf{halt} \mid \mathsf{recv}_c(x); P)) \mid \mathsf{recv}_c(x); Q$$

can only lead to the process $(\mathsf{new}\ c; P[M/x]) \mid \mathsf{recv}_c(x); Q$

Channels can also be sent as messages. Consider the following protocol where c_{AB} is a freshly created channel whereas c_{AS} and c_{SB} are long term channels.

$$A \longrightarrow S : c_{AB} \text{ on } c_{AS}$$

$$S \longrightarrow B : c_{AB} \text{ on } c_{SB}$$

$$A \longrightarrow B : M \text{ on } c_{AB}$$

can be represented as follows where $F(y)$ is a process involving variable y.

$$\begin{aligned} A &\triangleq \mathsf{new}\ c_{AB}; \mathsf{send}_{c_{AS}}\langle c_{AB}\rangle; \mathsf{send}_{c_{AB}}\langle M\rangle.\mathsf{halt} \\ S &\triangleq \mathsf{recv}_{c_{AS}}(x); \mathsf{send}_{c_{SB}}\langle x\rangle; \mathsf{halt} \\ B &\triangleq \mathsf{recv}_{c_{SB}}(x); \mathsf{recv}_{x}(y); F(y) \\ P &\triangleq \mathsf{new}\ c_{AS}; \mathsf{new}\ c_{SB}; (A \mid S \mid B) \end{aligned}$$

P makes silent transitions to $\mathsf{new}\ c_{AS}; \mathsf{new}\ c_{SB}; F(M)$.

Processes can perform computations like

- encryption, decryption (we will deal with only symmetric key encryption)
- pairing, unpairing
- increments, decrements
- checking equality of messages

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The process

$$\mathsf{recv}_c(x_1, x_2, x_3); \mathsf{case}\ x_1\ \mathsf{of}$$
$$\{y_1\}_K : \mathsf{check}\ (y_1 == x_2); \mathsf{send}_c\langle y_1, \mathsf{succ}\ (x_3)\rangle; \mathsf{halt}$$

receives an input of the form $\{M\}_K, M, N$ on channel c and sends out $y_1, \mathsf{succ}\ (x_3)$ on channel c.

The syntax

$M ::=$	term
n	name
(M, N)	pair
0	zero
$\mathsf{succ}\ (M)$	successor
$\{M_1, \ldots, M_k\}_N$	encryption
x	variable

$P ::=$	process
$\mathsf{send}_M\langle N_1, \ldots, N_k\rangle; P$	output
$\mathsf{recv}_M(x_1, \ldots, x_k); P$	input
halt	halt
$P \mid Q$	parallel composition
$\mathsf{repeat}\ P$	replication
$\mathsf{new}\ n; P$	restriction
$\mathsf{check}\ (M == N); P$	comparison
$\mathsf{let}\ (x, y) = M; P$	unpairing
$\mathsf{case}\ M\ \mathsf{of}\ 0 : P,\ \mathsf{succ}\ (x) : Q$	integer case analysis
$\mathsf{case}\ M\ \mathsf{of}\ \{x_1, \ldots, x_k\}_N : P$	decryption

Intuitively, $\mathsf{repeat}\ P$ represents infinitely many copies of P running in parallel.

In other words we can consider $\mathsf{repeat}\ P$ to represent $P \mid P \mid P \mid \ldots$

Consider

$$P \triangleq \mathsf{recv}_c(x); \mathsf{halt}$$

$$P_1 \triangleq \mathsf{send}_c(M_1); \mathsf{halt}$$

$$P_2 \triangleq \mathsf{send}_c(M_2); \mathsf{halt}$$

The process

$$P_1 \mid P_2 \mid \mathsf{repeat}\ P$$

can make silent transitions (internal communication) to create the process

$$\mathsf{repeat}\ P$$

A one message protocol using cryptography, where K_{AB} is a symmetric key shared between A and B for private communication.

$$A \longrightarrow B : \{M\}_{K_{AB}} \text{ on } c_{AB}$$

This can be represented as

$$\begin{aligned} A &\triangleq \mathsf{send}_{c_{AB}}\langle\{M\}_{K_{AB}}\rangle; \mathsf{halt} \\ B &\triangleq \mathsf{recv}_{c_{AB}}(x); \mathsf{case}\ x\ \mathsf{of}\ \{y\}_{K_{AB}} : F(y) \\ P &\triangleq \mathsf{new}\ K_{AB}; (A \mid B) \end{aligned}$$

The key K_{AB} is restricted, only A and B can use it.

The channel c_{AB} is public. Other principals may send messages on it or listen on it.

P can make silent transitions to $\mathsf{new}\ K_{AB}; F(M)$.

Formal semantics

We now need to define how processes execute.

For example we would like

$$\mathsf{send}_c\langle M\rangle; P \mid \mathsf{recv}_c(x); Q \xrightarrow{\tau} P \mid Q[M/x]$$

where τ denotes a silent action (internal communication).

Let $fn(M)$ and $fn(P)$ be the set of free names in term M and process P respectively.

Let $fv(M)$ and $fv(P)$ be the set of free variables in term M and process P respectively.

Closed processes are processes without any free variables.

Let $P \triangleq \mathsf{new}\ c; \mathsf{new}\ K; \mathsf{recv}_d(x); \mathsf{case}\ x\ \mathsf{of}\ \{y\}_{K'} : \mathsf{send}_d\langle\{y\}_K, z, c\rangle; \mathsf{halt}$.

We have

$fn(\mathsf{send}_d\langle\{y\}_K, z, c\rangle; \mathsf{halt}) = \{c, d, K\}$

$fv(\mathsf{send}_d\langle\{y\}_K, z, c\rangle; \mathsf{halt}) = \{y, z\}$

$fn(\mathsf{case}\ x\ \mathsf{of}\ \{y\}_{K'} : \mathsf{send}_d\langle\{y\}_K, z, c\rangle; \mathsf{halt}) = \{c, d, K, K'\}$

$fv(\mathsf{case}\ x\ \mathsf{of}\ \{y\}_{K'} : \mathsf{send}_d\langle\{y\}_K, z, c\rangle; \mathsf{halt}) = \{x, z\}$

$fn(P) = \{d, K'\}$

$fv(P) = \{z\}$

$fn(\{y\}_K) = \{K\}$

$fv(\{y\}_K) = \{y\}$

First we define reduction relation $>$ on closed processes:

$$
\begin{array}{rll}
\mathsf{repeat}\ P & > P \mid \mathsf{repeat}\ P & \text{(R-Repeat)} \\
\mathsf{check}\ (M == M); P & > P & \text{(R-Check)} \\
\mathsf{let}\ (x, y) = (M, N); P & > P[M/x, N/y] & \text{(R-Let)} \\
\mathsf{case}\ 0\ \mathsf{of}\ 0 : P,\ \mathsf{succ}\ (x) : Q & > P & \text{(R-Zero)} \\
\mathsf{case}\ \mathsf{succ}\ (M)\ \mathsf{of}\ 0 : P,\ \mathsf{succ}\ (x) : Q & > Q[M/x] & \text{(R-Succ)} \\
\mathsf{case}\ \{M\}_N\ \mathsf{of}\ \{x\}_N : P & > P[M/x] & \text{(R-decrypt)}
\end{array}
$$

When these rules cannot be applied, it means that the process cannot be simplified.

The following processes cannot be simplified, hence cannot be executed further.

$\mathsf{check}\ (0 == \mathsf{succ}\ (0); P$ (comparison fails).

$\mathsf{let}\ (x, y) = 0; P$ (unpairing fails)

$\mathsf{case}\ (M, N)\ \mathsf{of}\ 0 : P,\ \mathsf{succ}\ (x) : Q$ (not an integer)

$\mathsf{case}\ (M, N)\ \mathsf{of}\ \{x, y\}_K : P$ (not an encrypted message)

$\mathsf{case}\ \{M, N\}_{K'}\ \mathsf{of}\ \{x, y\}_K : P$ where $K \neq K'$

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This is also based on the perfect cryptography assumption: distinct terms represent distinct messages.

A barb β is either

- a name n (representing input on channel n), or
- a co-name $\overline{n}$ (representing output on channel n)

An action is either

- a barb (representing input or output to the outside world), or
- τ (representing a silent action i.e. internal communication)

We write $P \xrightarrow{\alpha} Q$ to mean that P makes action α after which Q is the remaining process that is left to be executed.

Commitment relation Consider again $\mathsf{send}_c\langle M\rangle; P \mid \mathsf{recv}_c(x); Q$

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The first subprocess makes an output action on channel c.

We will represent it as $\mathsf{send}_c\langle M\rangle; P \xrightarrow{\overline{c}} \langle M\rangle P$.

$\langle M\rangle P$ is called a concretion: it represents a commitment to output message M after which P will be executed.

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The second subprocess makes an input action on channel c.

We will represent it as $\mathsf{recv}_c(x); Q \xrightarrow{c} (x)Q$.

$(x)Q$ is called an abstraction:it represents a commitment to input some x after which P will be executed.

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Abstractions and concretions can be combined:

$\langle M\rangle P \;@\; (x)Q = P \mid Q[M/x]$