Formally an abstraction F is of the form

$$(x_1, \ldots, x_k)P$$

where $k \geq 0$ and P is a process.

A concretion C is of the form

$$(\mathsf{new}\ n_1, \ldots, n_l)\langle M_1, \ldots, M_k\rangle P$$

where $n_1, \ldots, n_l$ are names, $l, k \geq 0$ and P is a process.

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For $F \triangleq (x_1, \ldots, x_k)P$ and $C \triangleq (\mathsf{new}\ n_1, \ldots, n_l)\langle M_1, \ldots, M_k\rangle Q$

with $\{n_1, \ldots, n_l\} \cap fn(P) = \emptyset$ we define interaction of F and C as

$$\boxed{\begin{array}{l} F \ @\ C \triangleq \mathsf{new}\ n_1; \ldots \mathsf{new}\ n_l; (P[M_1/x_1, \ldots, M_k/x_k] \mid Q) \\ C \ @\ F \triangleq \mathsf{new}\ n_1; \ldots \mathsf{new}\ n_l; (Q \mid P[M_1/x_1, \ldots, M_k/x_k]) \end{array}}$$

An agent A is an abstraction, concretion or a process.

We write the commitment relation as $P \xrightarrow{\alpha} A$ where P is a closed process, A is a closed agent ($fv(A) = \emptyset$) and α is an action.

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$$\frac{P \xrightarrow{m} F \quad Q \xrightarrow{\overline{m}} C}{P \mid Q \xrightarrow{\tau} F @ C} \text{(C-Inter1)}$$

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$$\frac{P \xrightarrow{\overline{m}} C \quad Q \xrightarrow{m} F}{P \mid Q \xrightarrow{\tau} C \mathbin{@} F}\ \text{(C-Inter2)}$$

Example

Define

$P \triangleq \mathsf{send}_c\langle \mathsf{succ}\ (0)\rangle; \mathsf{halt}$

$Q \triangleq \mathsf{recv}_c(x); \mathsf{case}\ x\ \mathsf{of}\ 0 : \mathsf{halt},\ \mathsf{succ}\ (y) : (\mathsf{send}_d\langle y\rangle; \mathsf{halt})$

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$$P \quad \xrightarrow{\bar{c}} \langle \mathsf{succ}\ (0)\rangle \mathsf{halt}$$

$$(\langle M_1, \ldots, M_k\rangle P' \text{ denotes } (\mathsf{new}\ )\langle M_1, \ldots, M_k\rangle P')$$

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$$P \mid Q \xrightarrow{\tau} \mathsf{halt} \mid \mathsf{case}\ \mathsf{succ}\ (0)\ \mathsf{of}\ 0 : \mathsf{halt},\ \mathsf{succ}\ (y) : (\mathsf{send}_d\langle y\rangle; \mathsf{halt})$$

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$$\xrightarrow{\bar{d}} \langle 0\rangle(\mathsf{halt} \mid \mathsf{halt}) \quad \text{using the following rules\ldots}$$

$$\frac{P > Q \quad Q \xrightarrow{\alpha} A}{P \xrightarrow{\alpha} A} \text{ (C-Red)}$$

$$\frac{P \xrightarrow{\alpha} A}{P \mid Q \xrightarrow{\alpha} A \mid Q} \text{ (C-Par1)} \qquad \frac{Q \xrightarrow{\alpha} A}{P \mid Q \xrightarrow{\alpha} P \mid A} \text{ (C-Par2)}$$

where

$$P_1 \mid (x_1, \ldots, x_k)P_2 \triangleq (x_1, \ldots, x_k)(P_1 \mid P_2)$$

$$P_1 \mid (\mathsf{new}\ n_1, \ldots, n_k)\langle M_1, \ldots, M_l\rangle P_2 \triangleq (\mathsf{new}\ n_1, \ldots, n_k)\langle M_1, \ldots, M_l\rangle(P_1 \mid P_2)$$

provided that $x_1, \ldots, x_k \notin fv(P_1)$ and $n_1, \ldots, n_k \notin fn(P_1)$

For the previous example we have using (R-Succ):

$$\mathsf{case}\ \mathsf{succ}\ (0)\ \mathsf{of}\ 0 : \mathsf{halt},\ \mathsf{succ}\ (y) : (\mathsf{send}_d\langle y\rangle; \mathsf{halt}) > \mathsf{send}_d\langle 0\rangle; \mathsf{halt}$$

and using (C-Out):

$$\mathsf{send}_d\langle 0\rangle; \mathsf{halt} \xrightarrow{\overline{d}} \langle 0\rangle \mathsf{halt}$$

hence using (C-Red):

$$\mathsf{case}\ \mathsf{succ}\ (0)\ \mathsf{of}\ 0 : \mathsf{halt},\ \mathsf{succ}\ (y) : (\mathsf{send}_d\langle y\rangle; \mathsf{halt}) \xrightarrow{\overline{d}} \langle 0\rangle \mathsf{halt}$$

hence using (C-Par2):

$$\begin{aligned} \mathsf{halt} \mid \mathsf{case}\ \mathsf{succ}\ (0)\ \mathsf{of}\ 0 : \mathsf{halt},\ \mathsf{succ}\ (y) : (\mathsf{send}_d\langle y\rangle; \mathsf{halt}) &\xrightarrow{\overline{d}} \mathsf{halt} \mid \langle 0\rangle \mathsf{halt} \\ &= \langle 0\rangle(\mathsf{halt} \mid \mathsf{halt}) \end{aligned}$$

Consider $P \triangleq (\mathsf{recv}_c(x); P_1) \mid \mathsf{new}\ c; (\mathsf{send}_c\langle 0\rangle; P_2 \mid \mathsf{recv}_c(x); P_3)$

We would like $P \xrightarrow{\tau} (\mathsf{recv}_c(x); P_1) \mid \mathsf{new}\ c; (P_2 \mid P_3[0/x])$

but not $P \xrightarrow{\tau} P_1[0/x] \mid \mathsf{new}\ n; (P_2 \mid \mathsf{recv}_c(x); P_3)$

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Hence we have the rule

$$\boxed{\frac{P \xrightarrow{\alpha} A \quad \alpha \notin \{n, \overline{n}\}}{\mathsf{new}\ n; P \xrightarrow{\alpha} \mathsf{new}\ n; A}\ \text{(C-New)}}$$

where

$$(\mathsf{new}\ m)(x_1, \ldots, x_k)P \triangleq (x_1, \ldots, x_k)\mathsf{new}\ m; P$$

$$(\mathsf{new}\ m)(\mathsf{new}\ m_1, \ldots, m_k)\langle M_1, \ldots, M_l\rangle P \triangleq (\mathsf{new}\ m, m_1, \ldots, m_k)\langle M_1, \ldots, M_l\rangle P$$

provided that $m \notin \{m_1, \ldots, m_k\}$

We have $\mathsf{send}_c\langle 0\rangle; P_2 \xrightarrow{\overline{c}} \langle 0\rangle P_2$

and $\mathsf{recv}_c(x); P_3 \xrightarrow{c} (x)P_3$

hence $\mathsf{send}_c\langle 0\rangle; P_2 \mid \mathsf{recv}_c(x); P_3 \xrightarrow{\tau} \langle 0\rangle P_2 \mathbin{@} (x)P_3 = P_2 \mid P_3[0/x]$

Since $\tau \notin \{\overline{c}, c\}$

hence $\mathsf{new}\ c; (\mathsf{send}_c\langle 0\rangle; P_2 \mid \mathsf{recv}_c(x); P_3) \xrightarrow{\tau} \mathsf{new}\ c; (P_2 \mid P_3[0/x])$

Hence $(\mathsf{recv}_c(x); P_1) \mid \mathsf{new}\ c; (\mathsf{send}_c\langle 0\rangle; P_2 \mid \mathsf{recv}_c(x); P_3)$

$$\xrightarrow{\tau} (\mathsf{recv}_c(x); P_1) \mid \mathsf{new}\ c; (P_2 \mid P_3[0/x])$$

Consider $P \triangleq (\mathsf{new}\ K; \mathsf{send}_c\langle K\rangle; \mathsf{halt}) \mid (\mathsf{recv}_c(x); \mathsf{send}_d\langle x\rangle; \mathsf{halt})$

We have $\mathsf{send}_c\langle K\rangle; \mathsf{halt} \xrightarrow{\overline{c}} (\mathsf{new}\ )\langle K\rangle\mathsf{halt}$

hence $\mathsf{new}\ K; \mathsf{send}_c\langle K\rangle; \mathsf{halt} \xrightarrow{\overline{c}} \mathsf{new}\ K; (\mathsf{new}\ )\langle K\rangle\mathsf{halt} = (\mathsf{new}\ K)\langle K\rangle\mathsf{halt}$

Also $\mathsf{recv}_c(x); \mathsf{send}_d\langle x\rangle; \mathsf{halt} \xrightarrow{c} (x)\mathsf{send}_d\langle x\rangle; \mathsf{halt}$

Hence
$P \xrightarrow{\tau} (\mathsf{new}\ K)\langle K\rangle\mathsf{halt} \ @\ (x)\mathsf{send}_d\langle x\rangle; \mathsf{halt} = (\mathsf{new}\ K)(\mathsf{halt} \mid \mathsf{send}_d\langle K\rangle; \mathsf{halt})$

Equivalence on processes

A test is of the form (Q, β) where Q is a closed process and β is a barb.

A process P passes the test (Q, β) iff

$$(P \mid Q) \xrightarrow{\tau} Q_1 \ldots \xrightarrow{\tau} Q_n \xrightarrow{\beta} A$$

for some $n \geq 0$, some processes $Q_1, \ldots, Q_n$ and some agent A.

Q is the "environment" and we test whether the process together with the environment inputs or outputs on a particular channel.

Testing preorder $P_1 \sqsubseteq P_2$ iff for every test (Q, β), if P_1 passes (Q, β) then P_2 passes (Q, β).

Testing equivalence $P_1 \simeq P_2$ iff $P_1 \sqsubseteq P_2$ and $P_2 \sqsubseteq P_1$.

Secrecy

Consider process P with only free variable x.

We will consider x as secret if for all terms M, M' we have $P[M/x] \simeq P[M'/x]$.

I.e. an observer cannot detect any changes in the value of x.

Example Consider $P \triangleq \mathsf{send}_c\langle x\rangle; \mathsf{halt}$.

x is being sent out on a public channel. Consider test $(Q, \overline{d})$ where

environment $Q \triangleq \mathsf{recv}_c(x); \mathsf{check}\ (x == 0); \mathsf{send}_d\langle \mathsf{halt}\rangle; \mathsf{halt}$.

We have $P[0/x] \mid Q \xrightarrow{\tau} \mathsf{halt} \mid \mathsf{send}_d\langle 0\rangle; \mathsf{halt} \xrightarrow{\overline{d}} \langle 0\rangle(\mathsf{halt} \mid \mathsf{halt})$.

Hence $P[0/x]$ passes the test. However $P[\mathsf{succ}\ (0)/x]$ fails the test.

Hence P does not preserve secrecy of x.

Information flow analysis for the Spi-calculus

We classify data into three classes

secret	data which should not be leaked
public	data which can be communicated to anyone
any	arbitrary data

Subsumption relation on classes:

$$\begin{array}{lll}
\mathsf{secret} & \preceq \mathsf{any} & \\
\mathsf{public} & \preceq \mathsf{any} & \\
T & \preceq T & \text{for } T \in \{\mathsf{secret}, \mathsf{public}, \mathsf{any}\}
\end{array}$$

An environment E provides information about the classes to which names and variables belong.

We define typing rules for the following kinds of judgments

	$\vdash E$	environment E is well formed
E	$\vdash M : T$	term M is of class T in environment E
E	$\vdash P$	process P is well typed in environment E

E.g. secret data should not be sent on public channels.

Data of level any should be protected as if it is of level secret, but can be exploited only as of it had level public.

Our goal is to define typing rules to filter out processes that leak secrets.

Informally we would like to show that if environment E has only any variables and public names and $E \vdash P$ then P does not leak any variables $x \in dom(E)$.

Our previous example:

$P \triangleq \mathsf{send}_c\langle x\rangle; \mathsf{halt}$

Consider $E = \{x : \mathsf{any}, c : \mathsf{public} :: L_1, d : \mathsf{public} :: L_2\}$

(L_1 and L_2 will be explained later.)

x is of level any but is sent out on c of level public, which will be forbidden by our typing rules.

Consider protocol

$A \longrightarrow S : A, B$
$S \longrightarrow A : \{A, B, Na, \{Nb\}_{K_{sb}}\}_{K_{sa}}$
$A \longrightarrow B : \{Nb\}_{K_sb}$

A principal X may play the role of A in one session and of B in another session.

We need a clear way of distinguishing the messages received and their components.

This is important only for messages sent on secret channels and for messages encrypted with public keys.

We adopt the following standard format:

messages sent on secret channels should have three components of levels secret, any and public respectively.

Consider protocol

$B \longrightarrow A : Nb$

$A \longrightarrow B : \{M, Nb\}_{K_{ab}}$

By replaying nonces, an attacker can find out whether the same M is sent more than once, or different ones. Hence he gets

some partial information about the contents of the messages.

To prevent this we include an extra fresh nonce (confounder) in each message encrypted with secret keys.

$A \longrightarrow B : \{M, Nb, Na\}_{K_{ab}}$

We adopt the following standard format for messages encrypted with secret keys: $\{M_1, M_2, M_3, n\}_K$

where M_1 has level secret, M_2 has level any, M_3 has level public,

and n is the confounder.

n can be used as confounder only in this term and nowhere else.

This information is remembered by the environment E.

I.e. if $n : T :: \{M_1, M_2, M_3, n\}_K \in E$ then

we know that n is used as a confounder only in that message.

The typing rules

The empty environment is denoted $\emptyset$.

Well formed environments:

$$\vdash \emptyset$$

$$\frac{\vdash E \qquad x \notin dom(E)}{\vdash E, x : T}$$

$$\frac{\begin{array}{cc} \vdash E & n \notin dom(E) \\ E \vdash M_1 : T_1 \ldots E \vdash M_k : T_k & E \vdash N : R \end{array}}{\vdash E, n : T :: \{M_1, \ldots, M_k, n\}_N}$$

Environment lookups and subsumption:

$$\frac{E \vdash M : T \qquad T \sqsubseteq R}{E \vdash M : R}$$

$$\frac{\vdash E \qquad x : T \in E}{E \vdash x : T}$$

$$\frac{\vdash E \qquad n : T :: \{M_1, \ldots, M_k, n\}_N \in E}{E \vdash n : T}$$

$$\frac{\vdash E}{E \vdash 0 : \mathsf{public}}$$

$$\frac{E \vdash M : T}{E \vdash \mathsf{succ}\ (M) : T}$$

$$\frac{E \vdash M : T \qquad E \vdash N : T}{E \vdash \langle M, N \rangle : T}$$

Encryption

$$\frac{E \vdash M_1 : T \quad \ldots \quad E \vdash M_k : T \quad E \vdash N : \mathsf{public} \quad T = \mathsf{public} \text{ if } k = 0}{E \vdash \{M_1, \ldots, M_k\}_N : T}$$

$$\frac{\begin{array}{ccc} E \vdash M_1 : \mathsf{secret} & E \vdash M_2 : \mathsf{any} & E \vdash M_3 : \mathsf{public} \\ E \vdash N : \mathsf{secret} & & n : T :: \{M_1, M_2, M_3, n\}_N \in E \end{array}}{E \vdash \{M_1, M_2, M_3, n\}_N : \mathsf{public}}$$