# Virtual Machines 

## Summer Semester 2007

Deadline: 26 June 2007 12:00

## Exercise 1: <br> 6 Points

Write a prolog program including following predicates:
a) last/2 where the first parameter is a List and the second one is the last element of this list (e.g. last( $[1,2,3], 3))$.
b) reverse $/ 2$ with two lists as parameters, where one is the reverse list of the other. (e.g. reverse([1,2,3],[3,2,1]))
c) chain/2 with two lists, where the first list includes the second one as connected chain.
(e.g. chain $([1,2,3,4,5],[2,3,4]))$

Note: You can write auxiliary predicates if needed.

## Exercise 2:

5 Points
Produce $\operatorname{code}_{A} /$ code $_{G}$ for the following terms/goals !
a) $f(X, g(b, Y), g(\bar{X}, \bar{Z}))$
b) $p(f(g(X, h(\bar{Y},-), b), Z))$

Use the following address environment: $\rho=\{X \mapsto 1, Y \mapsto 2, Z \mapsto 3\}$ !
Exercise 3:
9 Points
Most General Unifiers: When two terms are to be unified, they are compared. If they are both constants then the result of unification is success if they are equal else failure. If they are both variables then one is bound to the other and unification succeds, If one is a variable $X$ and the other some structure $t$ then if $X$ occurs in $t$ then unification fails, otherwise $X$ is bound to $t$ and unification succeeds. If both terms are structures then each pair of sub-terms is unified recursively and the unification succeeds if each pair of sub-terms unifies.
The result of unification is either failure or success with a set of variable bindings, known as a "unifier". There may be many such unifiers for any pair of terms but there will be at most one "most general unifier", other unifiers simply add extra bindings for sub-terms which are variables in the original terms.

Determine the most general unifiers for the following pairs of terms if possible or explain why the unification fails:
a) $f(X, g(Y, b))$ and $f(g(a, Z), X)$
b) $f(g(a, Z), X)$ and $f(X, g(b, Y))$
c) $g(X, f(X, X))$ and $g(f(a, a), f(Y, Y))$
d) $g(g(X, g(a, Z)), g(f(V), V))$ and $g(g(f(Y), Y), g(Z, g(Y, Z)))$
e) $a(b, X, d(e, Y, g(h, i, Z)))$ and $a(U, c, d(V, f, g(W, i, j)))$
f) $f(X, 5, Y, x(a, g(6,7)))$ and $f(Y, 5, c, x(Z, g(6, X)))$

Example: The most general unifier of $f(X, g(Y, b), Z)$ and $f(g(a, U), g(a, V), W)$ is $[X / g(a, U), Y / a, Z / W, V / b]$. The terms $f(X, Y)$ and $f(g(Y), g(X))$ have no unifiers.

