The instruction setbtp saves the registers HP, TP, BP:


$$
\begin{aligned}
& \text { HPold = HP; } \\
& \text { TPold = TP; } \\
& \text { BPold = BP; } \\
& \text { BP = FP; }
\end{aligned}
$$

The instruction try A tries the alternative at address A and updates the negative continuation address to the current PC:


$$
\begin{aligned}
& \text { negForts = PC; } \\
& \text { PC = A; }
\end{aligned}
$$

The instruction delbtp restores the old backtrack pointer:

BP = BPold;

### 32.4 Popping of Stack Frames

Recall the translation scheme for clauses:

$$
\begin{aligned}
\operatorname{code}_{C} r= & \text { pushenv } \mathrm{m} \\
& \operatorname{code}_{G} g_{1} \rho \\
& \ldots \\
& \operatorname{code}_{G} g_{n} \rho \\
& \text { popenv }
\end{aligned}
$$

The present stack frame can be popped ...

- if the applied clause was the last (or only); and
- if all goals in the body are definitely finished.
$\Longrightarrow \quad$ the backtrack point is older :-)
$\Longrightarrow \quad \mathrm{FP}>\mathrm{BP}$

The instruction popenv restores the registers FP and PC and possibly pops the stack frame:


Warning: popenv may fail to de-allocate the frame !!!


If popping the stack frame fails, new data are allocated on top of the stack. When returning to the frame, the locals still can be accessed through the FP :-))

## 33 Queries and Programs

The translation of a program: $\quad p \equiv r r_{1} \ldots r r_{h} ? g$ consists of:

- an instruction no for failure;
- code for evaluating the query $g$;
- code for the predicate definitions $r r_{i}$.

Preceding query evaluation:
$\Longrightarrow \quad$ initialization of registers
$\Longrightarrow$ allocation of space for the globals

Succeeding query evaluation:
$\Longrightarrow$ returning the values of globals

$$
\begin{array}{ll}
\text { code } p=\quad & \text { init A } \\
& \text { pushenv d } \\
& \operatorname{code}_{G} g \rho \\
& \text { halt d } \\
\mathrm{A}: & \text { no } \\
& \operatorname{code}_{P} r r_{1} \\
& \ldots \\
& \operatorname{code}_{P} r r_{h}
\end{array}
$$

where $\operatorname{free}(g)=\left\{X_{1}, \ldots, X_{d}\right\} \quad$ and $\rho$ is given by $\rho X_{i}=i$.

The instruction halt d ...

- ... terminates the program execution;
- ... returns the bindings of the $d$ globals;
- ... causes backtracking - if demanded by the user :-)

The instruction init A is defined by:


At address " $A$ " for a failing goal we have placed the instruction no for printing no to the standard output and halt :-)

## The Final Example:

$$
\begin{array}{lll}
t(X) \leftarrow \bar{X}=b & q(X) \leftarrow s(\bar{X}) & s(X) \leftarrow \bar{X}=a \\
p \leftarrow q(X), t(\bar{X}) & s(X) \leftarrow t(\bar{X}) & ? p
\end{array}
$$

The translation yields:

|  | init N <br> pushenv 0 | $\mathrm{p} / 0$ : | popenv <br> pushenv 1 | $\mathrm{q} / 1$ : | pushenv 1 <br> mark D | E: | pushenv 1 <br> mark G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mark A |  | makr B |  | putref 1 |  | putref 1 |
|  | call p/0 |  | putvar 1 |  | call s/1 |  | call t/1 |
| A: | halt 0 |  | call q/1 | D: | popenv | G: | popenv |
| N : | no | B: | mark C | s/1: | setbtp | F: | pushenv 1 |
| t/1: | pushenv 1 |  | putref 1 |  | try E |  | putref 1 |
|  | putref 1 |  | call $\mathrm{t} / 1$ |  | delbtp |  | uatom a |
|  | uatom b | C: | popenv |  | jump F |  | popenv |

## 34 Last Call Optimization

Consider the app predicate from the beginnning:

$$
\begin{aligned}
& \operatorname{app}(X, Y, Z) \leftarrow X=[], Y=Z \\
& \operatorname{app}(X, Y, Z) \leftarrow X=\left[H \mid X^{\prime}\right], Z=\left[H \mid Z^{\prime}\right], \operatorname{app}\left(X^{\prime}, Y, Z^{\prime}\right)
\end{aligned}
$$

We observe:

- The recursive call occurs in the last goal of the clause.
- Such a goal is called last call.
$\Longrightarrow \quad$ we try to evaluate it in the current stack frame !!!
$\Longrightarrow$ after (successful) completion, we will not return to the current caller !!!

Consider a clause $r: \quad p\left(X_{1}, \ldots, X_{k}\right) \leftarrow g_{1}, \ldots, g_{n}$ with m locals where $\quad g_{n} \equiv q\left(t_{1}, \ldots, t_{h}\right)$. The interplay between code ${ }_{C}$ and code $_{G}$ :

$$
\operatorname{code}_{C} r=\quad \begin{aligned}
& \text { pushenv } \mathrm{m} \\
& \\
& \operatorname{code}_{G} g_{1} \rho \\
& \\
& \ldots \\
& \operatorname{code}_{G} g_{n-1} \rho \\
& \operatorname{mark~B~} \quad \\
& \operatorname{code}_{A} t_{1} \rho \\
& \\
& \ldots \\
& \\
& \operatorname{code}_{A} t_{h} \rho \\
& \\
& \operatorname{call~q}^{\prime} \mathrm{h} \\
& \mathrm{~B}: \\
& \text { popenv }
\end{aligned}
$$

| Replacement: | mark $B$ <br> call $\mathrm{q} / \mathrm{h} ;$ popenv | $\Longrightarrow$ | lastmark |
| :--- | :--- | :--- | :--- |
|  |  | lastcall $\mathrm{q} / \mathrm{h} \mathrm{m}$ |  |

Consider a clause $r$ :

$$
p\left(X_{1}, \ldots, X_{k}\right) \leftarrow g_{1}, \ldots, g_{n}
$$

with m locals where $\quad g_{n} \equiv q\left(t_{1}, \ldots, t_{h}\right)$. The interplay between code ${ }_{C}$ and $\operatorname{code}_{G}$ :

$$
\begin{aligned}
\operatorname{code}_{C} r= & \text { pushenv m } \\
& \operatorname{code}_{G} g_{1} \rho \\
& \ldots \\
& \operatorname{code}_{G} g_{n-1} \rho \\
& \text { lastmark } \\
& \operatorname{code}_{A} t_{1} \rho \\
& \ldots \\
& \operatorname{code}_{A} t_{h} \rho \\
& \text { lastcall }^{q} / \mathrm{h} \mathrm{~m}
\end{aligned}
$$

| Replacement: | mark B <br> call q/h; popenv | $\Longrightarrow$ | lastmark |
| :--- | :--- | :--- | :--- |
|  | $\Longrightarrow$ | lastcall q/h m |  |

If the current clause is not last or the $g_{1}, \ldots, g_{n-1}$ have created backtrack points, then $\mathrm{FP} \leq \mathrm{BP} \quad:-$ )

Then lastmark creates a new frame but stores a reference to the predecessor:


If $\mathrm{FP}>\mathrm{BP}$ then lastmark does nothing :-)

If $\mathrm{FP} \leq \mathrm{BP}$, then lastcall $\mathrm{q} / \mathrm{hm}$ behaves like a normal call $\mathrm{q} / \mathrm{h}$.
Otherwise, the current stack frame is re-used. This means that:

- the cells $\mathrm{S}[\mathrm{FP}+1], \mathrm{S}[\mathrm{FP}+2], \ldots, \mathrm{S}[\mathrm{FP}+\mathrm{h}]$ receive the new values and
- $q / h$ can be jumped to :-)

$$
\begin{aligned}
& \text { lastcall } \mathrm{q} / \mathrm{h} \mathrm{~m}=\quad \text { if }(\mathrm{FP} \leq \mathrm{BP}) \text { call } \mathrm{q} / \mathrm{h} ; \\
& \text { else }\{ \\
& \text { move } \mathrm{m} \mathrm{~h} ; \\
& \text { jump q/h; } \\
&\quad\}
\end{aligned}
$$

The difference between the old and the new addresses of the parameters m just equals the number of the local variables of the current clause :-))


## Example:

Consider the clause:

$$
\mathrm{a}(X, Y) \leftarrow \mathrm{f}\left(\bar{X}, X_{1}\right), \mathrm{a}\left(\bar{X}_{1}, \bar{Y}\right)
$$

The last-call optimization for $\quad \operatorname{code}_{C} r$ yields:

|  | mark A | A: | lastmark |
| :--- | :--- | :--- | :--- |
| pushenv 3 | putref 1 |  | putref 3 |
| putvar 3 |  | putref 2 |  |
|  | call f/2 |  | lastcall a/2 3 |

## Example:

Consider the clause:

$$
\mathrm{a}(X, Y) \leftarrow \mathrm{f}\left(\overline{\mathrm{X}}, \mathrm{X}_{1}\right), \mathrm{a}\left(\bar{X}_{1}, \bar{Y}\right)
$$

The last-call optimization for $\operatorname{codec}_{C} r$ yields:

|  | mark A | A: | lastmark |
| :--- | :--- | :--- | :--- |
| pushenv 3 | putref 1 |  | putref 3 |
| putvar 3 |  | putref 2 |  |
|  | call f/2 |  | lastcall a/2 3 |

Note:
If the clause is last and the last literal is the only one, we can skip lastmark and can replace lastcall $\mathrm{q} / \mathrm{h} \mathrm{m}$ with the sequence move m n ; jump $\mathrm{p} / \mathrm{n}$ :-))

## Example:

Consider the last clause of the app predicate:

$$
\operatorname{app}(X, Y, Z) \leftarrow \bar{X}=\left[H \mid X^{\prime}\right], \bar{Z}=\left[\bar{H} \mid Z^{\prime}\right], \operatorname{app}\left(\bar{X}^{\prime}, \bar{Y}, \bar{Z}^{\prime}\right)
$$

Here, the last call is the only one :-) Consequently, we obtain:

| A: | pushenv 6 |  |  |  | uref 4 |  | bind |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | putref 1 | B: | putvar 4 |  | son 2 | E | putref 5 |
|  | ustruct [l]/2 B |  | putvar 5 |  | uvar 6 |  | putref 2 |
|  | son 1 |  | putstruct [\|]/2 |  | up E |  | putref 6 |
|  | uvar 4 |  | bind | D: | check 4 |  | move 63 |
|  | son 2 | C: | putref 3 |  | putref 4 |  | jump app/3 |
|  | uvar 5 |  | ustruct [l]/2 D |  | putvar 6 |  |  |
|  | up C |  | son 1 |  | putstruct |  |  |

## 35 Trimming of Stack Frames

Idea:

- Order local variables according to their life times;
- Pop the dead variables - if possible :-\}


## 35 Trimming of Stack Frames

## Idea:

- Order local variables according to their life times;
- Pop the dead variables - if possible :-\}

Example:

Consider the clause:

$$
\mathrm{a}(X, Z) \leftarrow \mathrm{p}_{1}\left(\bar{X}, X_{1}\right), \mathrm{p}_{2}\left(\bar{X}_{1}, X_{2}\right), \mathrm{p}_{3}\left(\bar{X}_{2}, X_{3}\right), \mathrm{p}_{4}\left(\bar{X}_{3}, \bar{Z}\right)
$$

## 35 Trimming of Stack Frames

## Idea:

- Order local variables according to their life times;
- Pop the dead variables - if possible :-\}

Example:
Consider the clause:

$$
\mathrm{a}(X, Z) \leftarrow \mathrm{p}_{1}\left(\bar{X}, X_{1}\right), \mathrm{p}_{2}\left(\bar{X}_{1}, X_{2}\right), \mathrm{p}_{3}\left(\bar{X}_{2}, X_{3}\right), \mathrm{p}_{4}\left(\bar{X}_{3}, \bar{Z}\right)
$$

After the query $p_{2}\left(\bar{X}_{1}, X_{2}\right)$, variable $X_{1}$ is dead.
After the query $p_{3}\left(\bar{X}_{2}, X_{3}\right)$, variable $X_{2}$ is dead :-)

After every non-last goal with dead variables, we insert the instruction trim :


After every non-last goal with dead variables, we insert the instruction trim :

trim m


$$
\begin{aligned}
& \text { if }(\mathrm{FP} \geq \mathrm{BP}) \\
& \quad \mathrm{SP}=\mathrm{FP}+\mathrm{m} ;
\end{aligned}
$$

The dead locals can only be popped if no new backtrack point has been allocated :-)

## Example (continued):

$$
\mathrm{a}(X, Z) \leftarrow \mathrm{p}_{1}\left(\bar{X}, X_{1}\right), \mathrm{p}_{2}\left(\bar{X}_{1}, X_{2}\right), \mathrm{p}_{3}\left(\bar{X}_{2}, X_{3}\right), \mathrm{p}_{4}\left(\bar{X}_{3}, \bar{Z}\right)
$$

Ordering of the variables:

$$
\rho=\left\{X \mapsto 1, Z \mapsto 2, X_{3} \mapsto 3, X_{2} \mapsto 4, X_{1} \mapsto 5\right\}
$$

The resulting code:

| pushenv 5 | A: | mark B | mark C | lastmark |
| :---: | :---: | :---: | :---: | :---: |
| mark A |  | putref 5 | putref 4 | putref 3 |
| putref 1 |  | putvar 4 | putvar 3 | putref 2 |
| putvar 5 |  | call $\mathrm{p}_{2} / 2$ | call $\mathrm{p}_{3} / 2$ | lastcall $p_{4} / 23$ |
| call $\mathrm{p}_{1} / 2$ | B: | trim 4 | trim 3 |  |

## 36 Clause Indexing

## Observation:

Often, predicates are implemented by case distinction on the first argument.
$\Longrightarrow \quad$ Inspecting the first argument, many alternatives can be excluded :-)
$\Longrightarrow \quad$ Failure is earlier detected :-)
$\Longrightarrow \quad$ Backtrack points are earlier removed. :-))
$\Longrightarrow \quad$ Stack frames are earlier popped :-)))

Example: The app-predicate:

$$
\begin{aligned}
& \operatorname{app}(X, Y, Z) \leftarrow X=[], Y=Z \\
& \operatorname{app}(X, Y, Z) \leftarrow X=\left[H \mid X^{\prime}\right], Z=\left[H \mid Z^{\prime}\right], \operatorname{app}\left(X^{\prime}, Y, Z^{\prime}\right)
\end{aligned}
$$

- If the root constructor is [ ], only the first clause is applicable.
- If the root constructor is [|], only the second clause is applicable.
- Every other root constructor should fail !!
- Only if the first argument equals an unbound variable, both alternatives must be tried ;-)


## Idea:

- Introduce separate try chains for every possible constructor.
- Inspect the root node of the first argument.
- Depending on the result, perform an indexed jump to the appropriate try chain.

Assume that the predicate $\mathrm{p} / \mathrm{k}$ is defined by the sequence $r r$ of clauses $r_{1} \ldots r_{m}$. Let tchains $r$ denote the sequence of try chains as built up for the root constructors occurring in unifications $X_{1}=t$.

## Example:

Consider again the app-predicate, and assume that the code for the two clauses start at addresses $A_{1}$ and $A_{2}$, respectively.
Then we obtain the following four try chains:

VAR: setbtp // variables NIL: jump $A_{1}$ // atom []
try $A_{1}$
delbtp
jump $A_{2}$
CONS: jump $A_{2} \quad / /$ constructor [|]

ELSE: fail // default

## Example:

Consider again the app-predicate, and assume that the code for the two clauses start at addresses $A_{1}$ and $A_{2}$, respectively.
Then we obtain the following four try chains:

| VAR: | setbtp | // variables | NIL: | jump $A_{1}$ |
| :--- | :--- | :--- | :--- | :--- | // atom [ ]

The new instruction fail takes care of any constructor besides [ ] and [|] ...

$$
\text { fail }=\text { backtrack() }
$$

It directly triggers backtracking :-)

Then we generate for a predicate $p / k$ :

```
\mp@subsup{code}{P}{}rr = putref 1
    getNode // extracts the root label
    index p/k // jumps to the try block
    tchains rr
    A1: codec rr
    Am: codec r rm
```

The instruction getNode returns " $R$ " if the pointer on top of the stack points to an unbound variable. Otherwise, it returns the content of the heap object:

getNode


```
switch (H[S[SP]]) {
case (S, f/n): }\quad\textrm{S}[\textrm{SP}]=\textrm{f}/\textrm{n};\mathrm{ break;
case (A,a): }\quad\textrm{S}[\textrm{SP}]=a;\mathrm{ break;
case (R,_): }\quad\textrm{S}[\textrm{SP}]=\textrm{R}
}
```

The instruction index $\mathrm{p} / \mathrm{k}$ performs an indexed jump to the appropriate try chain:

index $\mathrm{p} / \mathrm{k}$


PC map (p/k,a)

$$
\begin{aligned}
& \mathrm{PC}=\operatorname{map}(\mathrm{p} / \mathrm{k}, \mathrm{~S}[\mathrm{SP}]) ; \\
& \mathrm{SP}--;
\end{aligned}
$$

The instruction index $\mathrm{p} / \mathrm{k}$ performs an indexed jump to the appropriate try chain:

| $a$ |
| ---: |
|  |
|  | index $\mathrm{p} / \mathrm{k}$



$$
\begin{aligned}
& \mathrm{PC}=\operatorname{map}(\mathrm{p} / \mathrm{k}, \mathrm{~S}[\mathrm{SP}]) ; \\
& \mathrm{SP}--;
\end{aligned}
$$

The function $\operatorname{map}()$ returns, for a given predicate and node content, the start address of the appropriate try chain :-)

It typically is defined through some hash table :-))

