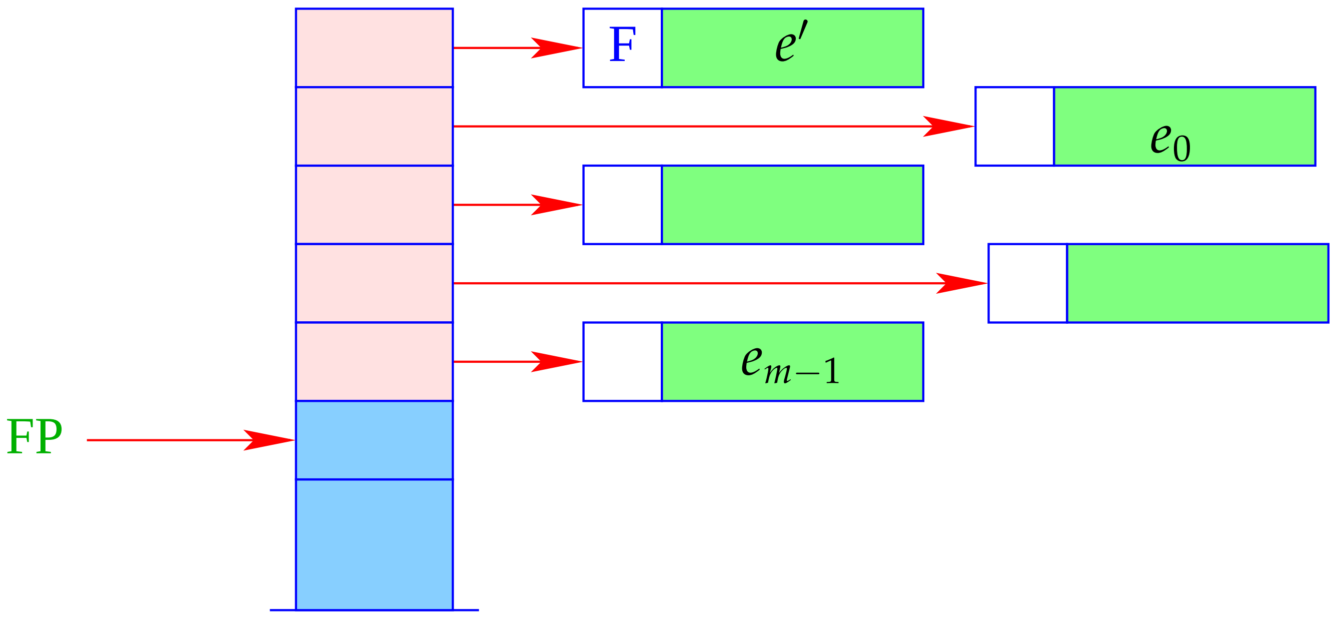
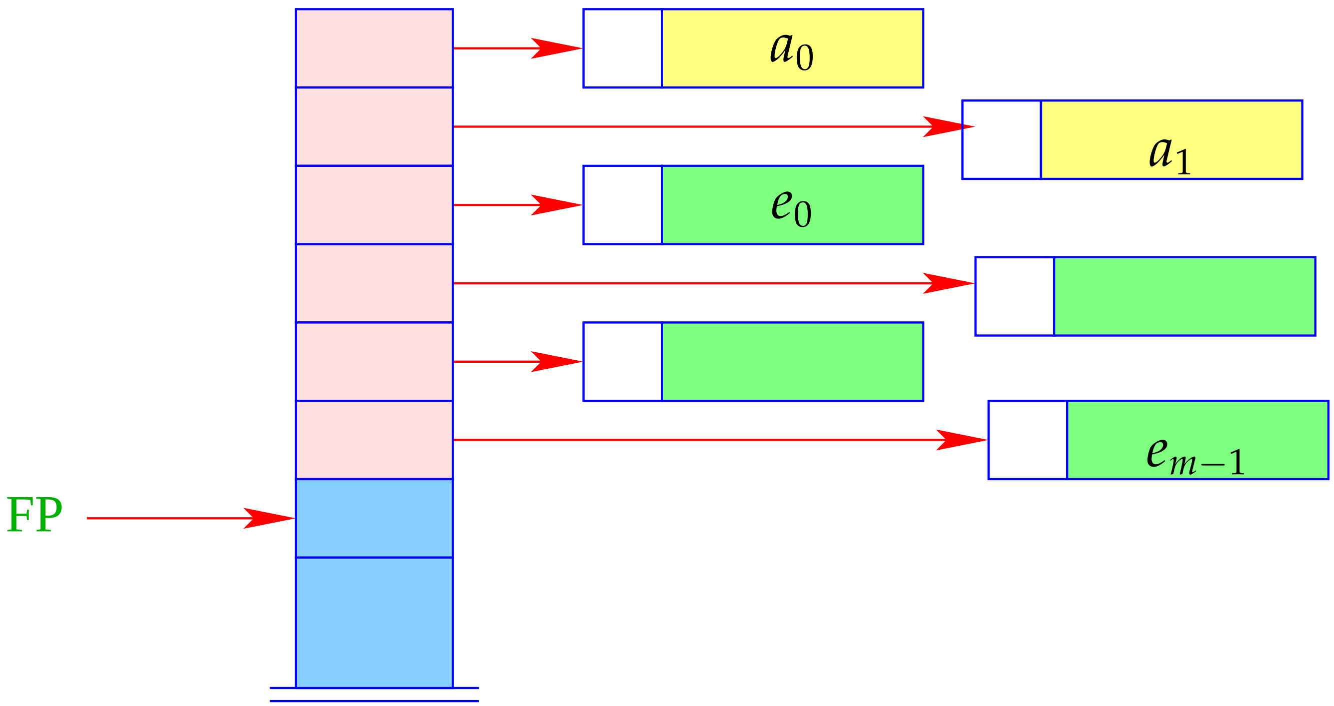
Alternative:



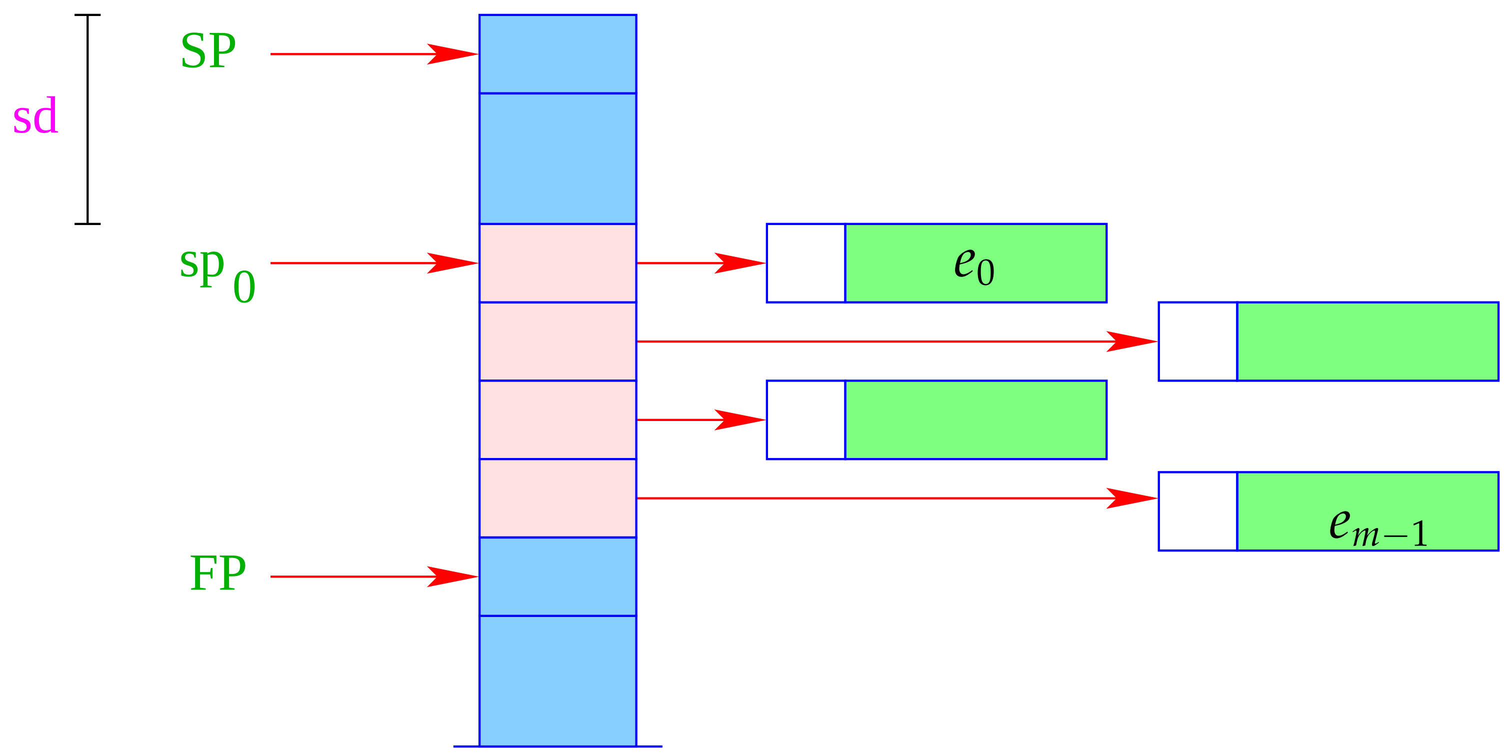
+ The further arguments $a_0, \ldots, a_{k-1}$ and the local variables can be allocated above the arguments.



– Addressing of arguments and local variables relative to FP is no more possible. (Remember: m is unknown when the function definition is translated.)

Way out:

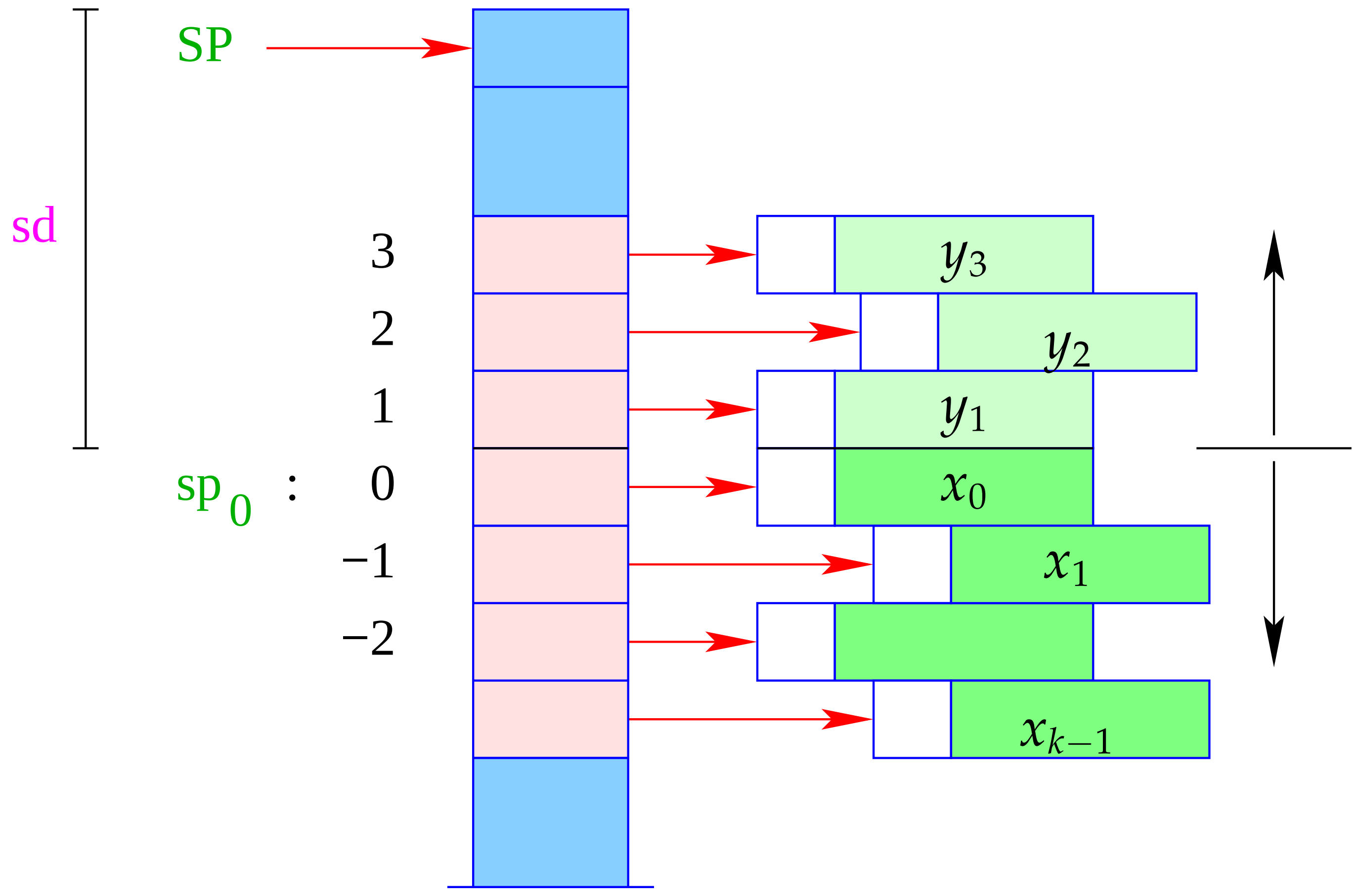
- We address both, arguments and local variables, relative to the stack pointer SP !!!
- However, the stack pointer changes during program execution...



- The differerence between the current value of SP and its value sp_0 at the entry of the function body is called the stack distance, sd.
- Fortunately, this stack distance can be determined at compile time for each program point, by simulating the movement of the SP.
- The formal parameters $x_0, x_1, x_2, \ldots$ successively receive the non-positive relative addresses $0, -1, -2, \ldots$, i.e., $\rho\, x_i = (L, -i)$.
- The absolute address of the i-th formal parameter consequently is

$$\mathrm{sp}_0 - i = (\mathrm{SP} - \mathrm{sd}) - i$$

- The local **let**-variables $y_1, y_2, y_3, \ldots$ will be successively pushed onto the stack:



- The y_i have positive relative addresses $1, 2, 3, \ldots$, that is: $\rho\ y_i = (L, i)$.
- The absolute address of y_i is then $\text{sp}_0 + i = (\text{SP} - \text{sd}) + i$

With CBN, we generate for the access to a variable:

$$\text{code}_V\ x\ \rho\ \text{sd} \quad = \quad \begin{array}{l} \text{getvar}\ x\ \rho\ \text{sd} \\ \text{eval} \end{array}$$

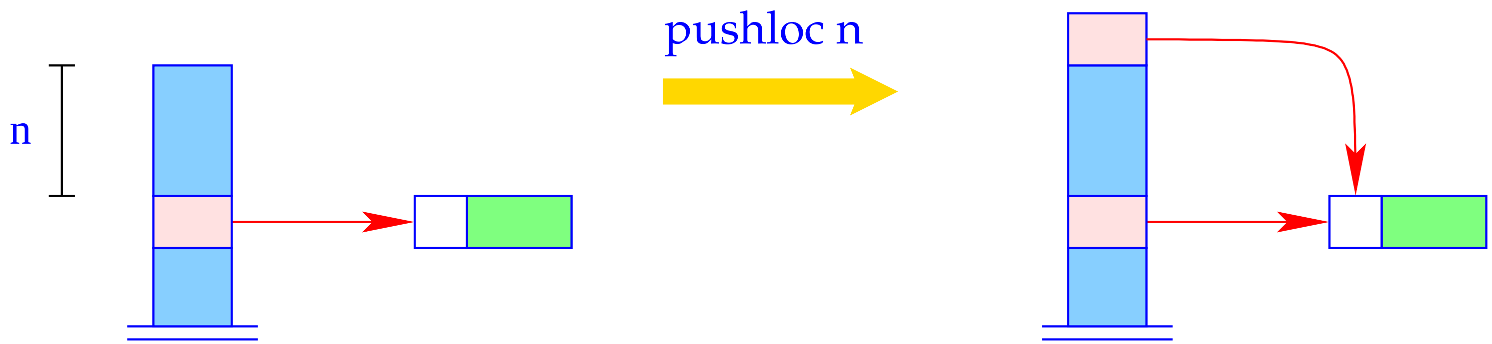
The instruction eval checks, whether the value has already been computed or whether its evaluation has to yet to be done ($\Longrightarrow$ will be treated later :-)

With CBV, we can just delete eval from the above code schema.

The (compile-time) macro getvar is defined by:

$$\begin{array}{lll} \text{getvar}\ x\ \rho\ \text{sd} & = & \textbf{let}\ (t, i) = \rho\ x\ \textbf{in} \\ & & \textbf{case}\ t\ \textbf{of} \\ & & \quad L \Rightarrow \text{pushloc}\ (\text{sd} - i) \\ & & \quad G \Rightarrow \text{pushglob i} \\ & & \textbf{end} \end{array}$$

The access to local variables:



S[SP+1] =S[SP - n]; SP++;

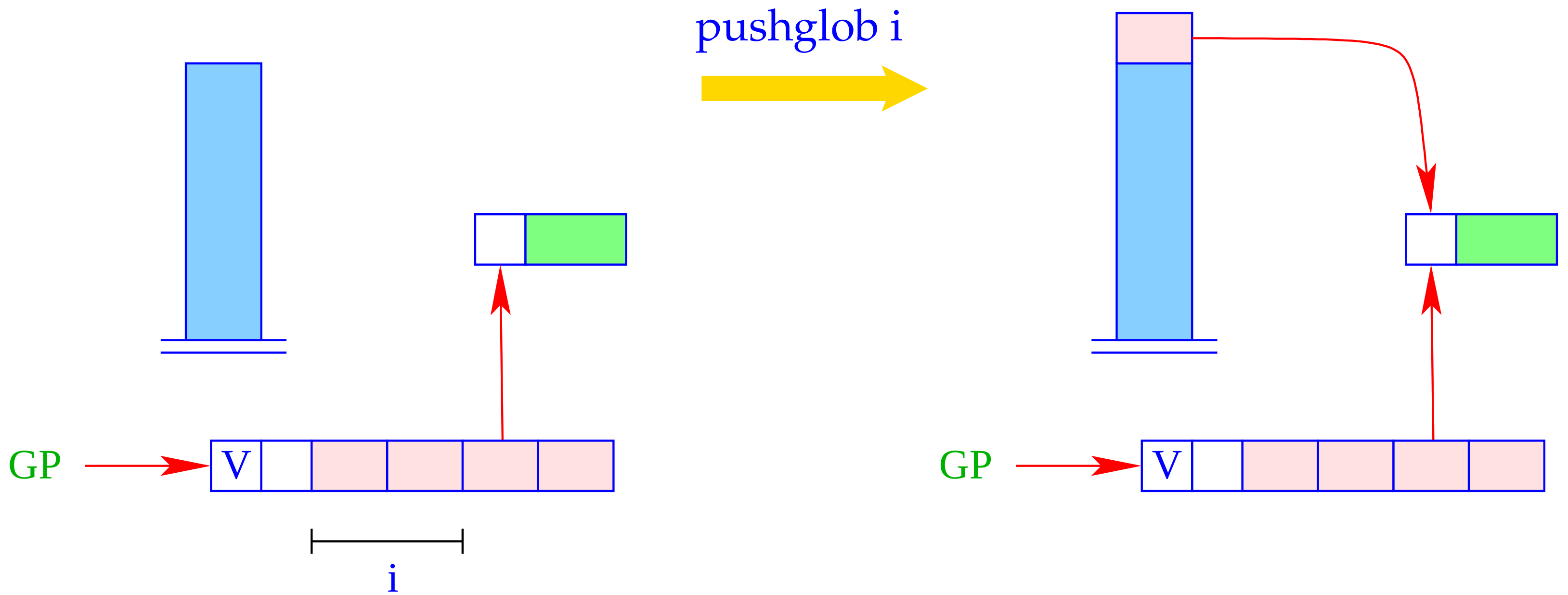
Correctness argument:

Let sp and sd be the values of the stack pointer resp. stack distance before the execution of the instruction. The value of the local variable with address i is loaded from $S[a]$ with

$$a = \mathrm{sp} - (\mathrm{sd} - i) = (\mathrm{sp} - \mathrm{sd}) + i = \mathrm{sp}_0 + i$$

... exactly as it should be :-)

The access to global variables is much simpler:



SP = SP + 1;
S[SP] = GP→v[i];

Example:

Regard $e \equiv (b + c)$ for $\rho = \{b \mapsto (L, 1), c \mapsto (G, 0)\}$ and $\mathsf{sd} = 1$.

With CBN, we obtain:

$$
\begin{array}{llllll}
\mathrm{code}_V\ e\ \rho\ 1 & = & \mathrm{getvar}\ b\ \rho\ 1 & = & 1 & \mathrm{pushloc}\ 0 \\
 & & \mathrm{eval} & & 2 & \mathrm{eval} \\
 & & \mathrm{getbasic} & & 2 & \mathrm{getbasic} \\
 & & \mathrm{getvar}\ c\ \rho\ 2 & & 2 & \mathrm{pushglob}\ 0 \\
 & & \mathrm{eval} & & 3 & \mathrm{eval} \\
 & & \mathrm{getbasic} & & 3 & \mathrm{getbasic} \\
 & & \mathrm{add} & & 3 & \mathrm{add} \\
 & & \mathrm{mkbasic} & & 2 & \mathrm{mkbasic}
\end{array}
$$

15 let-Expressions

As a warm-up let us first consider the treatment of local variables :-)

Let $e \equiv \textbf{let } y_1 = e_1; \ldots; y_n = e_n \textbf{ in } e_0$ be a **let**-expression.

The translation of e must deliver an instruction sequence that

- allocates local variables $y_1, \ldots, y_n$;
- in the case of
  CBV: evaluates $e_1, \ldots, e_n$ and binds the y_i to their values;
  CBN: constructs closures for the $e_1, \ldots, e_n$ and binds the y_i to them;
- evaluates the expression e_0 and returns its value.

Here, we consider the non-recursive case only, i.e. where y_j only depends on $y_1, \ldots, y_{j-1}$. We obtain for CBN:

$$
\begin{array}{rcll}
\text{code}_V\ e\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} & \\
 & & \text{code}_C\ e_2\ \rho_1\ (\text{sd}+1) & \\
 & & \ldots & \\
 & & \text{code}_C\ e_n\ \rho_{n-1}\ (\text{sd}+n-1) & \\
 & & \text{code}_V\ e_0\ \rho_n\ (\text{sd}+n) & \\
 & & \text{slide n} & \text{// deallocates local variables}
\end{array}
$$

where $\rho_j = \rho \oplus \{y_i \mapsto (L, \text{sd} + i) \mid i = 1, \ldots, j\}$.

In the case of CBV, we use code_V for the expressions $e_1, \ldots, e_n$.

Warning!

All the e_i must be associated with the same binding for the global variables!

Example:

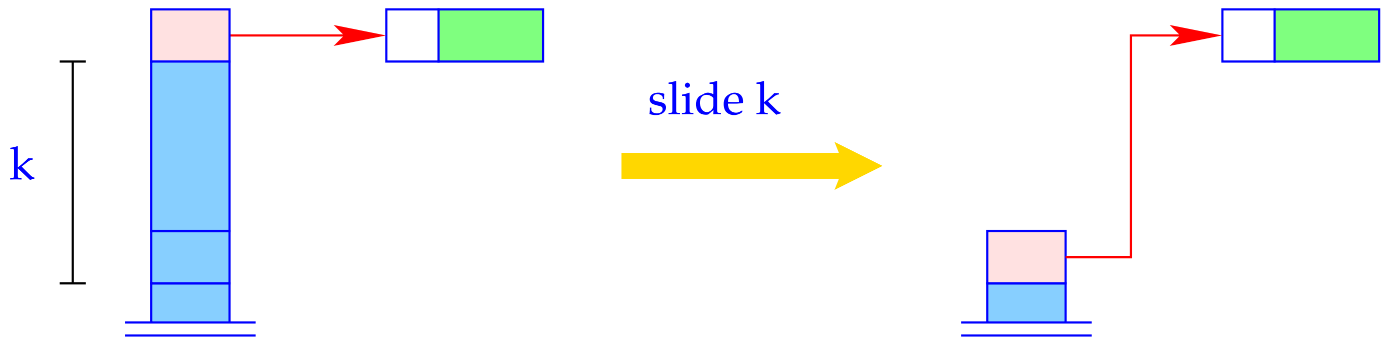
Consider the expression

$$e \equiv \textbf{let } a = 19; b = a * a \textbf{ in } a + b$$

for $\rho = \emptyset$ and sd $= 0$. We obtain (for CBV):

0	loadc 19	3	getbasic	3	pushloc 1
1	mkbasic	3	mul	4	getbasic
1	pushloc 0	2	mkbasic	4	add
2	getbasic	2	pushloc 1	3	mkbasic
2	pushloc 1	3	getbasic	3	slide 2

The instruction slide k deallocates again the space for the locals:



S[SP-k] = S[SP];
SP = SP - k;

16 Function Definitions

The definition of a function f requires code that allocates a functional value for f in the heap. This happens in the following steps:

- Creation of a Global Vector with the binding of the free variables;
- Creation of an (initially empty) argument vector;
- Creation of an F-Object, containing references to these vectors and the start address of the code for the body;

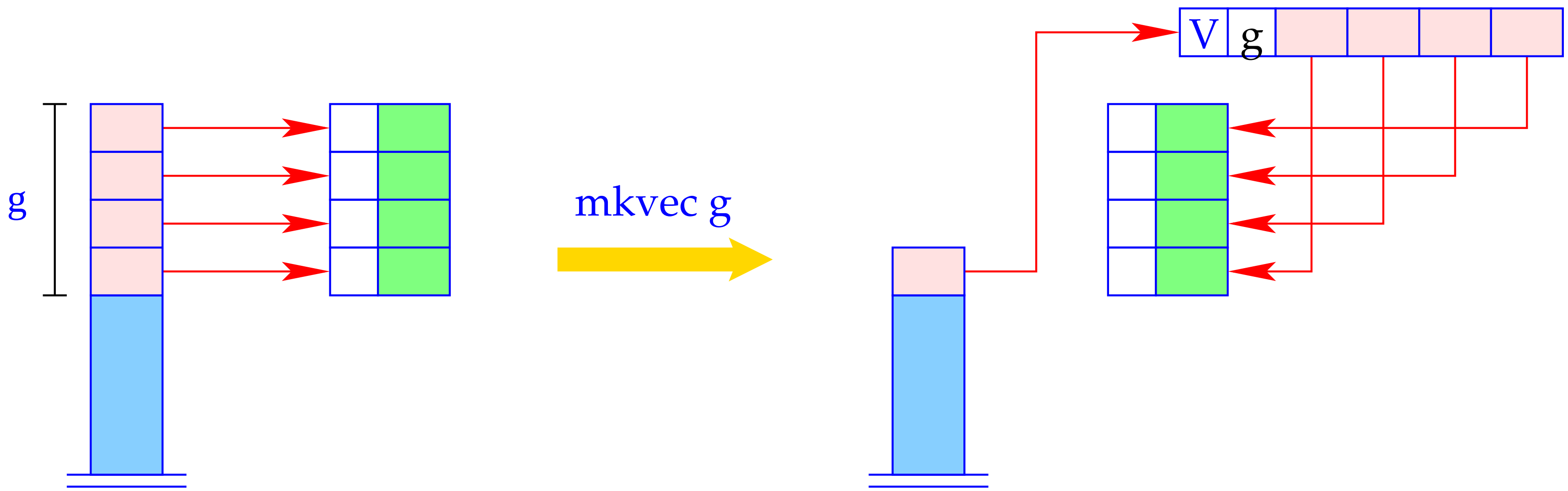
Separately, code for the body has to be generated.

Thus:

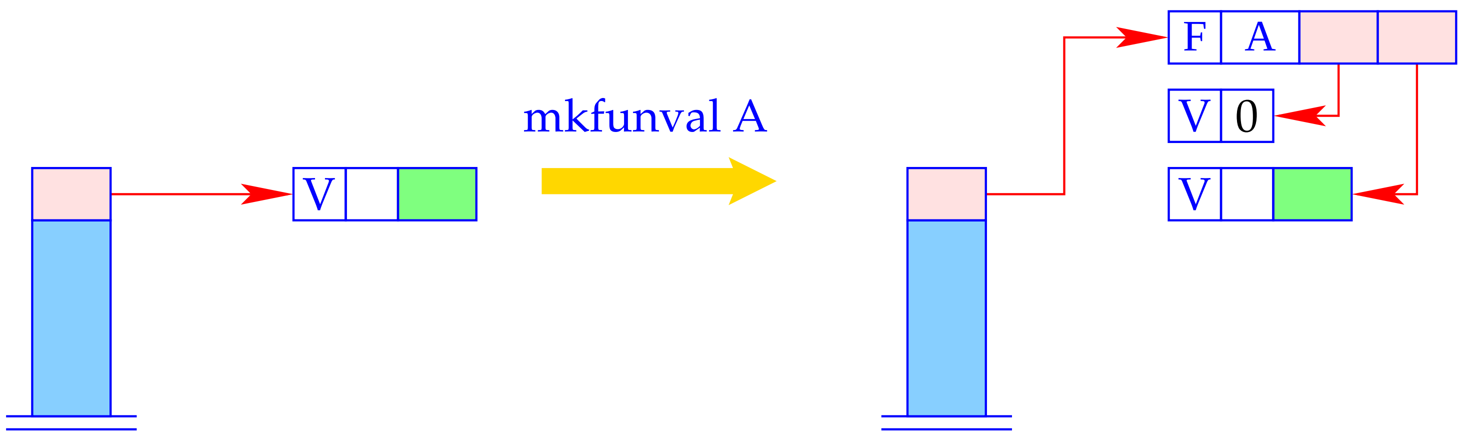
$$
\begin{array}{rlrl}
\mathrm{code}_V\ (\mathbf{fn}\ x_0,\ldots,x_{k-1} \Rightarrow e)\ \rho\ \mathrm{sd} & = & & \mathrm{getvar}\ z_0\ \rho\ \mathrm{sd} \\
& & & \mathrm{getvar}\ z_1\ \rho\ (\mathrm{sd}+1) \\
& & & \ldots \\
& & & \mathrm{getvar}\ z_{g-1}\ \rho\ (\mathrm{sd}+g-1) \\
& & & \mathrm{mkvec\ g} \\
& & & \mathrm{mkfunval\ A} \\
& & & \mathrm{jump\ B} \\
& & \mathrm{A}: & \mathrm{targ\ k} \\
& & & \mathrm{code}_V\ e\ \rho'\ 0 \\
& & & \mathrm{return\ k} \\
& & \mathrm{B}: & \ldots
\end{array}
$$

where $\{z_0,\ldots,z_{g-1}\} = \mathit{free}(\mathbf{fn}\ x_0,\ldots,x_{k-1} \Rightarrow e)$

and $\rho' = \{x_i \mapsto (L,-i) \mid i = 0,\ldots,k-1\} \cup \{z_j \mapsto (G,j) \mid j = 0,\ldots,g-1\}$




```
h = new (V, n);
SP = SP - g + 1;
for (i=0; i<g; i++)
    h→v[i] = S[SP + i];
S[SP] = h;
```



a = new (V,0);
S[SP] = new (F, A, a, S[SP]);

Example:

Regard $f \equiv \textbf{fn}\ b \Rightarrow a + b$ for $\rho = \{a \mapsto (L, 1)\}$ and $\text{sd} = 1$.

$\text{code}_V\ f\ \rho\ 1$ produces:

1		pushloc 0	0	pushglob 0	2		getbasic
2		mkvec 1	1	eval	2		add
2		mkfunval A	1	getbasic	1		mkbasic
2		jump B	1	pushloc 1	1		return 1
0	A :	targ 1	2	eval	2	B :	...

The secrets around targ k and return k will be revealed later :-)

17 Function Application

Function applications correspond to function calls in C.
The necessary actions for the evaluation of $e' \, e_0 \, \ldots \, e_{m-1}$ are:

- Allocation of a stack frame;
- Transfer of the actual parameters , i.e. with:
  CBV: Evaluation of the actual parameters;
  CBN: Allocation of closures for the actual parameters;
- Evaluation of the expression e' to an F-object;
- Application of the function.

Thus for CBN:

$$
\begin{array}{rcll}
\text{code}_V\ (e'\ e_0\ \ldots\ e_{m-1})\ \rho\ \text{sd} & = & \text{mark A} & \text{// Allocation of the frame} \\
 & & \text{code}_C\ e_{m-1}\ \rho\ (\text{sd}+3) & \\
 & & \text{code}_C\ e_{m-2}\ \rho\ (\text{sd}+4) & \\
 & & \ldots & \\
 & & \text{code}_C\ e_0\ \rho\ (\text{sd}+m+2) & \\
 & & \text{code}_V\ e'\ \rho\ (\text{sd}+m+3) & \text{// Evaluation of } e' \\
 & & \text{apply} & \text{// corresponds to call} \\
 & A: & \ldots &
\end{array}
$$

To implement CBV, we use code_V instead of code_C for the arguments e_i.

Example: For $(f\ 42)$, $\rho = \{f \mapsto (L, 2)\}$ and $\text{sd} = 2$, we obtain with CBV:

2	mark A	6	mkbasic	7		apply
5	loadc 42	6	pushloc 4	3	A :	...

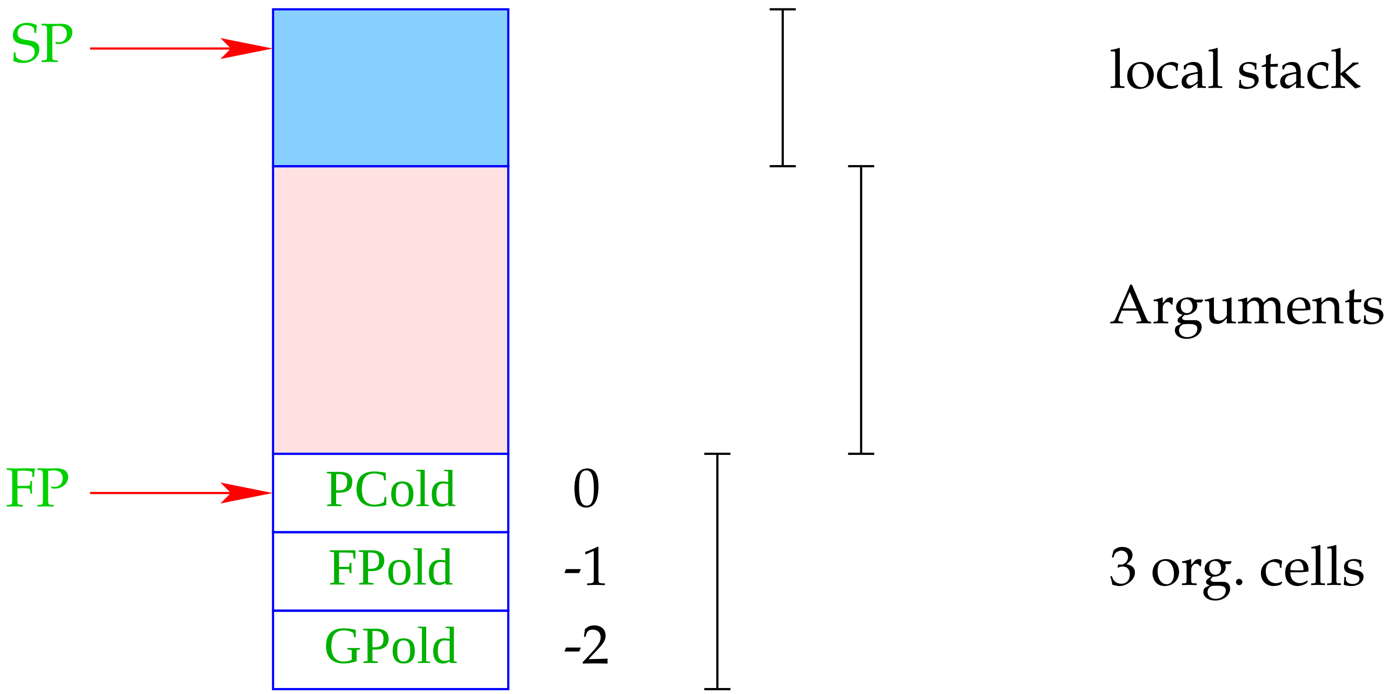
A Slightly Larger Example:

$$\mathbf{let}\ a = 17;\ f = \mathbf{fn}\ b \Rightarrow a + b\ \mathbf{in}\ f\ 42$$

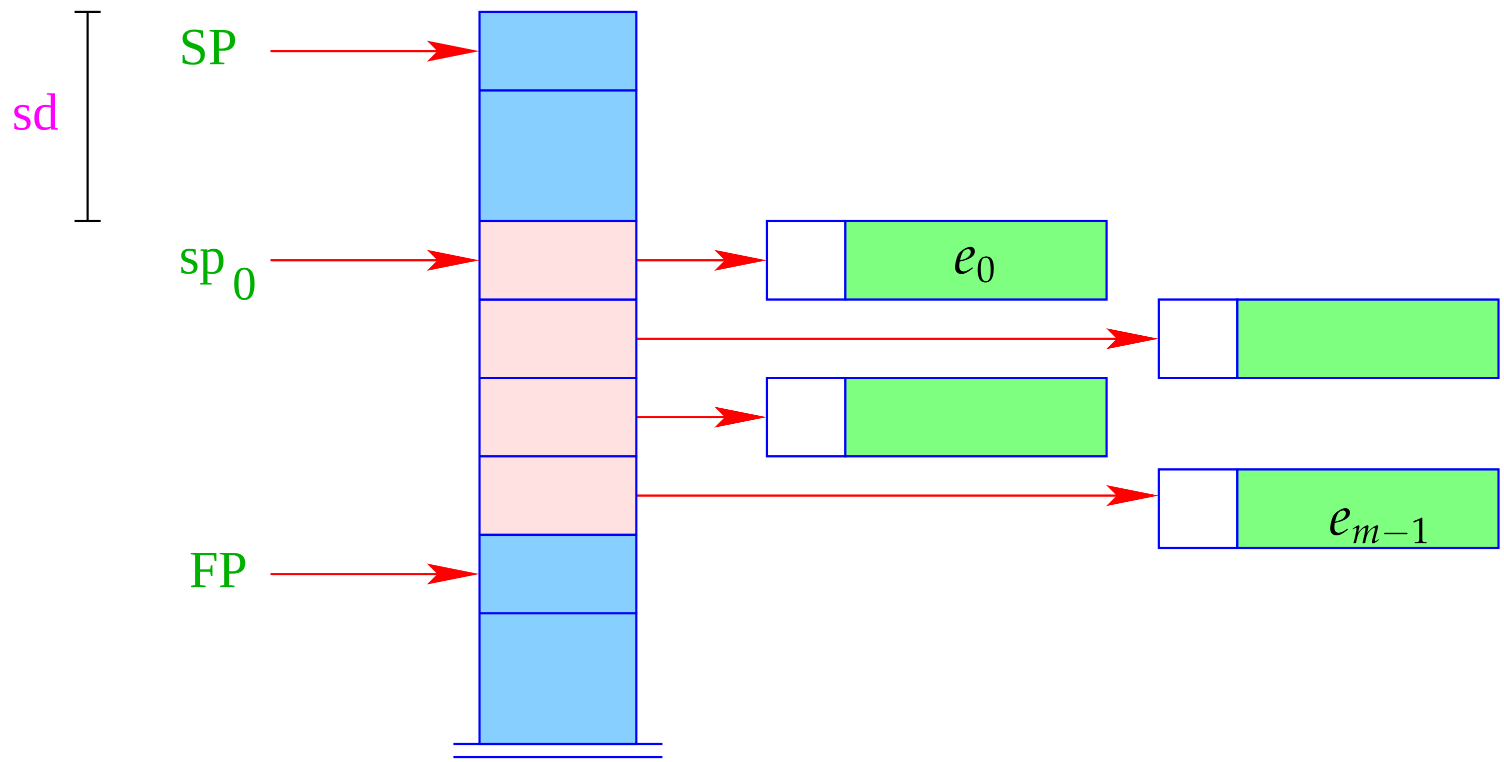
For CBV and $\quad \mathsf{sd} = 0 \quad$ we obtain:

0	loadc 17	2		jump B	2		getbasic	5		loadc 42
1	mkbasic	0	A:	targ 1	2		add	5		mkbasic
1	pushloc 0	0		pushglob 0	1		mkbasic	6		pushloc 4
2	mkvec 1	1		getbasic	1		return 1	7		apply
2	mkfunval A	1		pushloc 1	2	B:	mark C	3	C:	slide 2

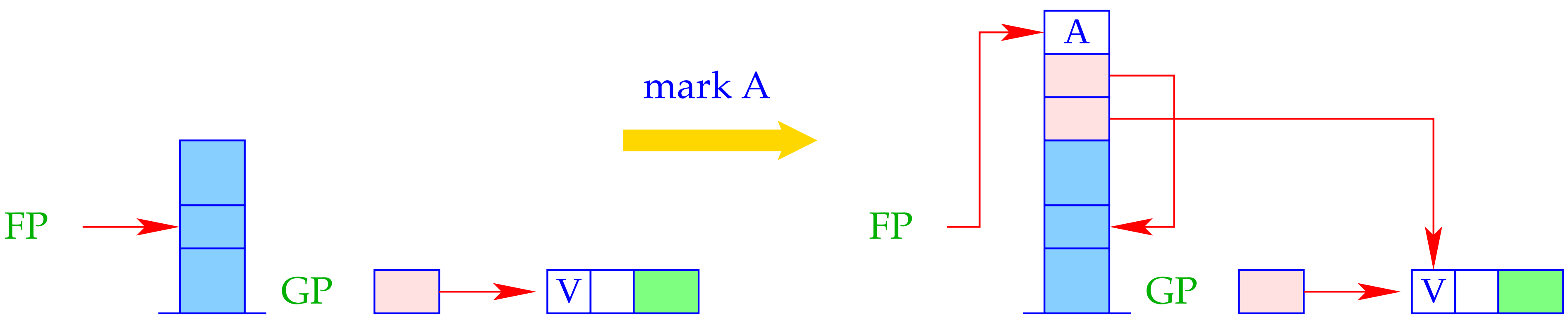
For the implementation of the new instruction, we must fix the organization of a stack frame:



Remember: Addressing of arguments and local variables

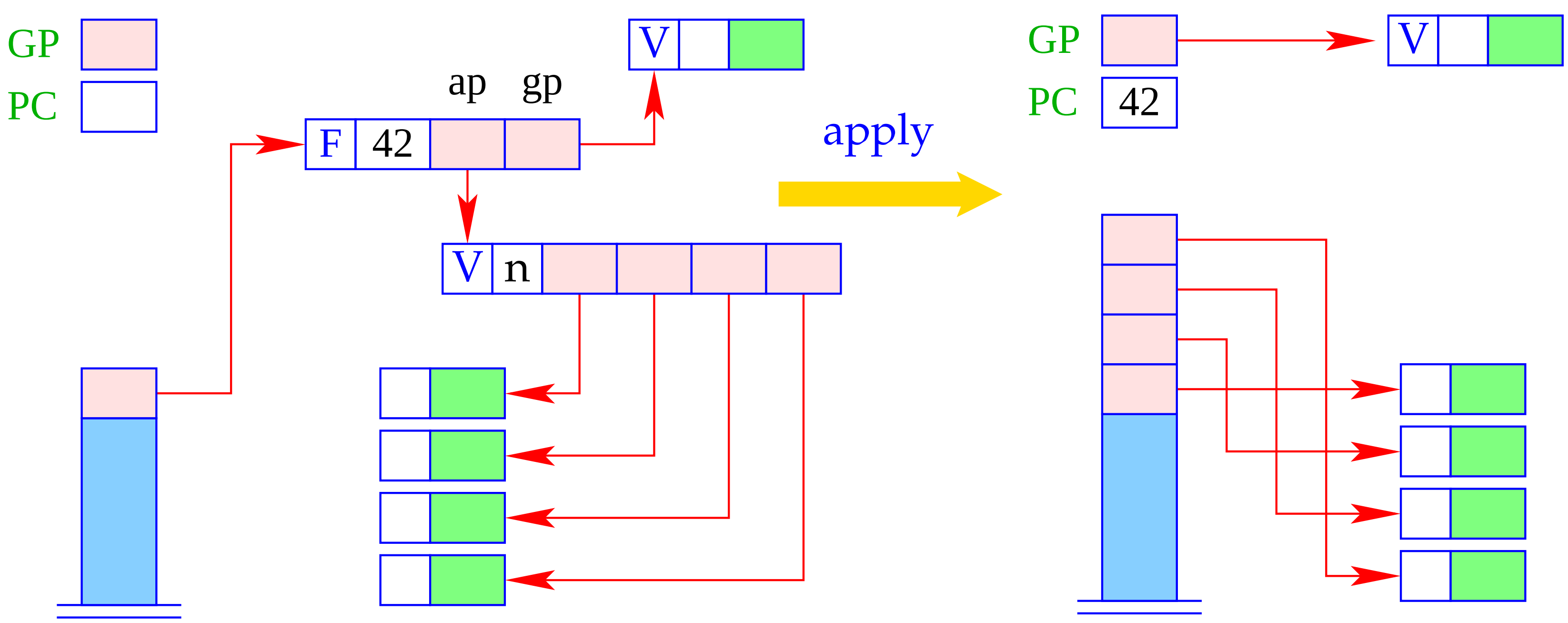


Different from the CMa, the instruction mark A already saves the return address:



S[SP+1] = GP;
S[SP+2] = FP;
S[SP+3] = A;
FP = SP = SP + 3;

The instruction apply unpacks the F-object, a reference to which (hopefully) resides on top of the stack, and continues execution at the address given there:



h = S[SP];
if (H[h] != (F,_,_))
    Error "no fun";
else {

GP = h→gp; PC = h→cp;
for (i=0; i< h→ap→n; i++)
    S[SP+i] = h→ap→v[i];
SP = SP + h→ap→n – 1;
}

Warning:

- The last element of the argument vector is the last to be put onto the stack. This must be the first argument reference.
- This should be kept in mind, when we treat the packing of arguments of an under-supplied function application into an F-object !!!

18 Over– and Undersupply of Arguments

The first instruction to be executed when entering a function body, i.e., after an apply is targ k .

This instruction checks whether there are enough arguments to evaluate the body.

Only if this is the case, the execution of the code for the body is started.

Otherwise, i.e. in the case of under-supply, a new F-object is returned.

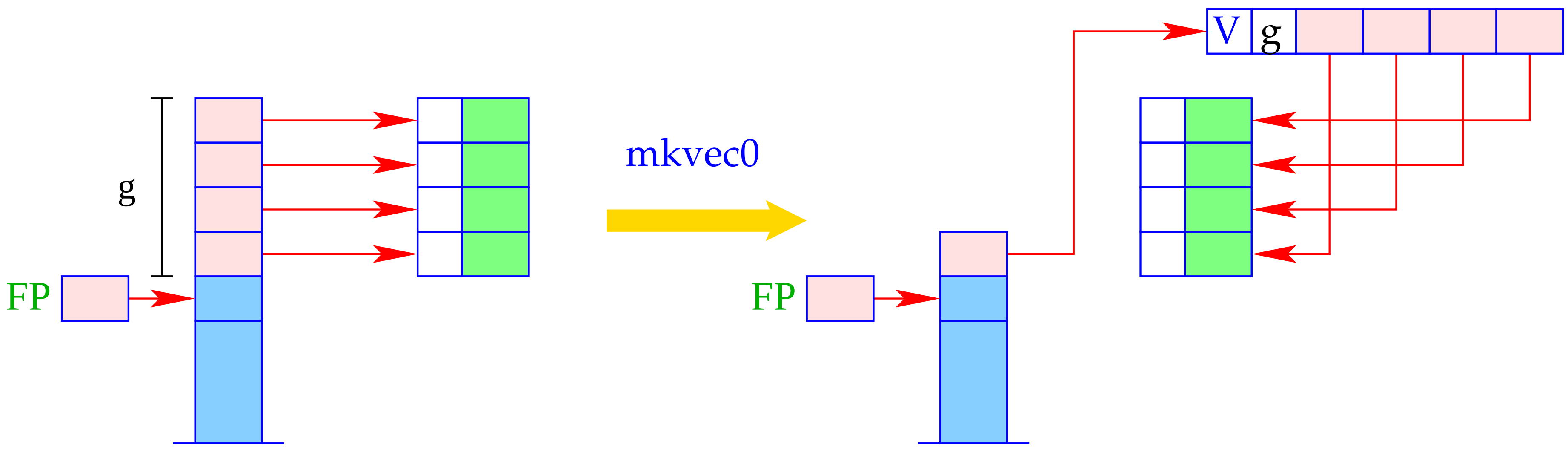
The test for number of arguments uses: SP – FP targ k   is a complex instruction.

We decompose its execution in the case of under-supply into several steps:

$$
\begin{array}{lll}
\text{targ k} & = & \text{if (SP – FP} < \text{k) \{} \\
 & & \quad \text{mkvec0;} \qquad\qquad // \ \text{creating the argumentvector} \\
 & & \quad \text{wrap;} \qquad\qquad\quad // \ \text{wrapping into an F} - \text{object} \\
 & & \quad \text{popenv;} \qquad\qquad // \ \text{popping the stack frame} \\
 & & \text{\}}
\end{array}
$$

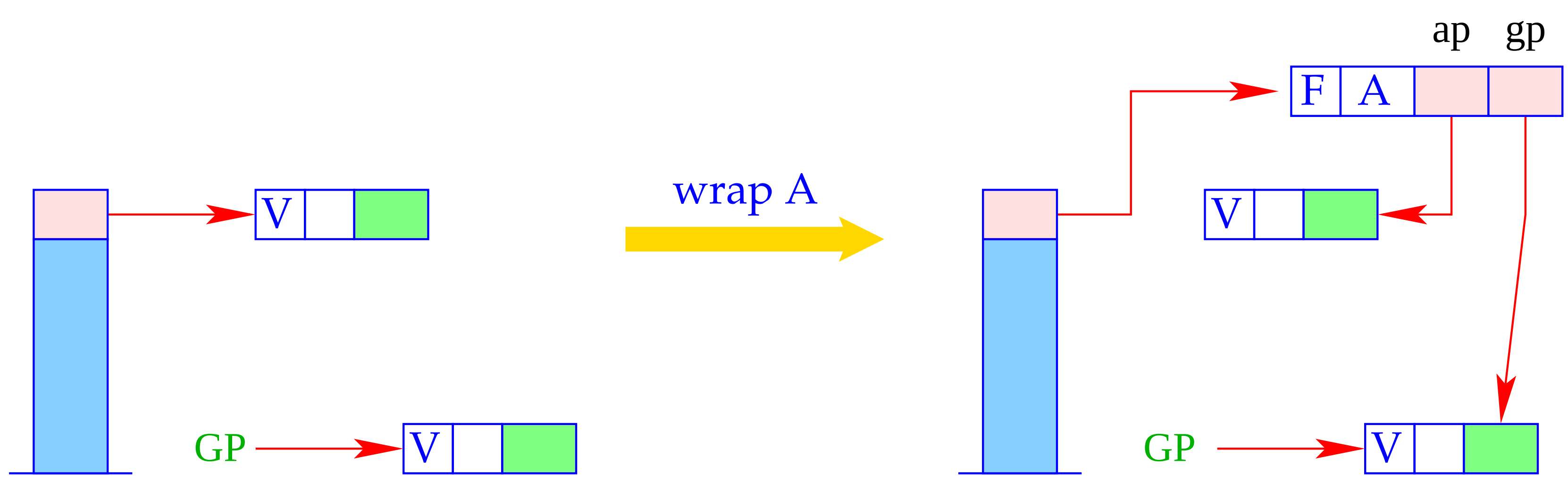
The combination of these steps into one instruction is a kind of optimization   :-)

The instruction mkvec0 takes all references from the stack above FP and stores them into a vector:



$g = SP - FP$; $h = $ new (V, g);
$SP = FP + 1$;
for $(i=0; i<g; i{+}{+})$
    $h \rightarrow v[i] = S[SP + i]$;
$S[SP] = h$;

The instruction wrap A wraps the argument vector together with the global vector into an F-object:



S[SP] = new (F, A, S[SP], GP);