21 Optimizations I: Global Variables

Observation:

- Functional programs construct many F- and C-objects.
- This requires the inclusion of (the bindings of) all global variables.

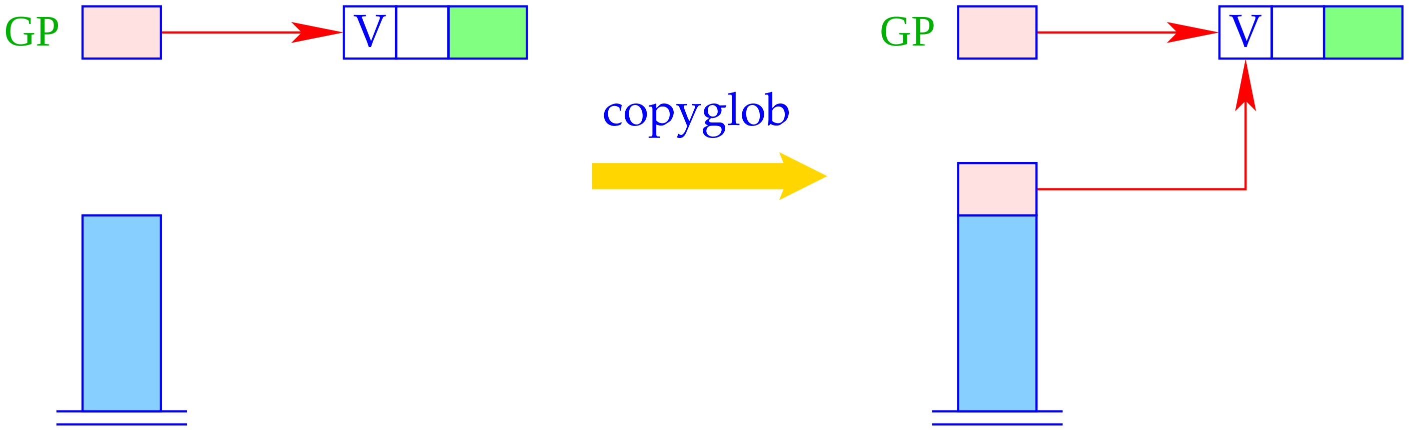
  Recall, e.g., the construction of a closure for an expression *e* ...

$$
\begin{array}{rcrl}
\mathrm{code}_C\ e\ \rho\ \mathrm{sd} & = & & \mathrm{getvar}\ z_0\ \rho\ \mathrm{sd} \\
 & & & \mathrm{getvar}\ z_1\ \rho\ (\mathrm{sd}+1) \\
 & & & \dots \\
 & & & \mathrm{getvar}\ z_{g-1}\ \rho\ (\mathrm{sd}+g-1) \\
 & & & \mathrm{mkvec}\ \mathrm{g} \\
 & & & \mathrm{mkclos}\ \mathrm{A} \\
 & & & \mathrm{jump}\ \mathrm{B} \\
 & & \mathrm{A}: & \mathrm{code}_V\ e\ \rho'\ 0 \\
 & & & \mathrm{update} \\
 & & \mathrm{B}: & \dots
\end{array}
$$

where $\{z_0, \dots, z_{g-1}\} = \mathit{free}(e)$ and $\rho' = \{z_i \mapsto (G, i) \mid i = 0, \dots, g-1\}$.

Idea:

- Reuse Global Vectors, i.e. share Global Vectors!
- Profitable in the translation of **let**-expressions or function applications: Build one Global Vector for the union of the free-variable sets of all let-definitions resp. all arguments.
- Allocate (references to) global vectors with multiple uses in the stack frame like local variables!
- Support the access to the current GP by an instruction copyglob :



SP++;
S[SP] = GP;

- The optimization will cause Global Vectors to contain more components than just references to the free the variables that occur in one expression ...

Disadvantage: Superfluous components in Global Vectors prevent the deallocation of already useless heap objects $\Longrightarrow$ Space Leaks :-(

Potential Remedy: Deletion of references at the end of their life time.

22 Optimizations II: Closures

In some cases, the construction of closures can be avoided, namely for

- Basic values,
- Variables,
- Functions.

Basic Values:

The construction of a closure for the value is at least as expensive as the construction of the B-object itself!

Therefore:

$$\begin{array}{lllll} \text{code}_C\ b\ \rho\ \text{sd} & = & \text{code}_V\ b\ \rho\ \text{sd} & = & \text{loadc b} \\ & & & & \text{mkbasic} \end{array}$$

This replaces:

```
    mkvec 0            jump B       mkbasic        B:    ...
    mkclos A       A:  loadc b      update
```

Variables:

Variables are either bound to values or to C-objects. Constructing another closure is therefore superfluous. Therefore:

$$\text{code}_C\; x\; \rho\; \text{sd} \quad = \quad \text{getvar}\; x\; \rho\; \text{sd}$$

This replaces:

```
    getvar x ρ sd
    mkvec 1
    mkclos A
    jump B
A:  pushglob 0
    eval
    update
B:  ...
```

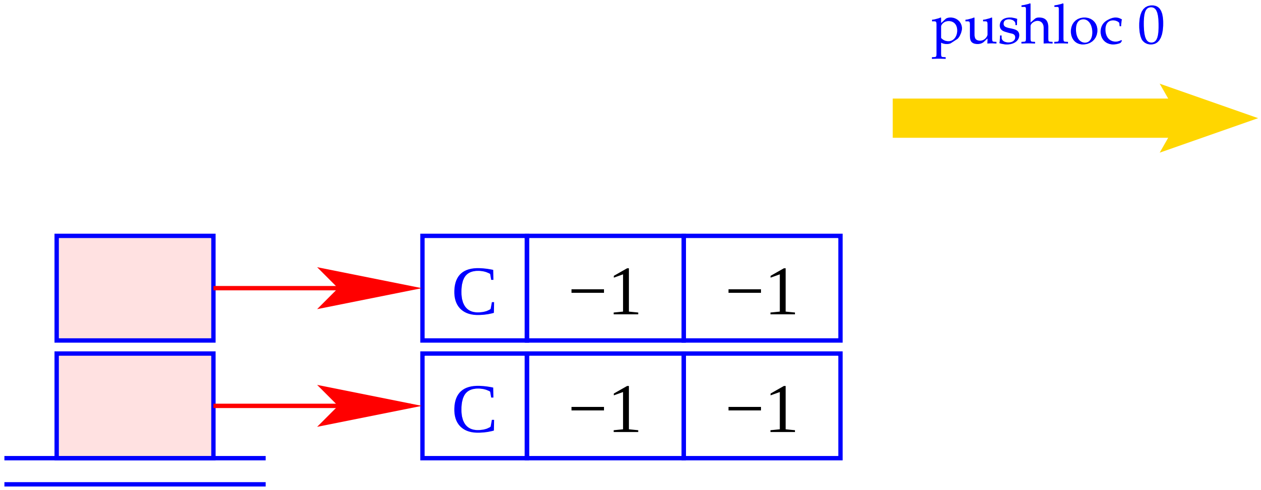
Example: $e \equiv \textbf{letrec}\; a = b; b = 7\; \textbf{in}\; a.$ $\quad \text{code}_V\; e\; \emptyset\; 0$ produces:

```
0   alloc 2
2   pushloc 0
3   rewrite 2
2   loadc 7
3   mkbasic
3   rewrite 1
2   pushloc 1
3   eval
3   slide 2
```

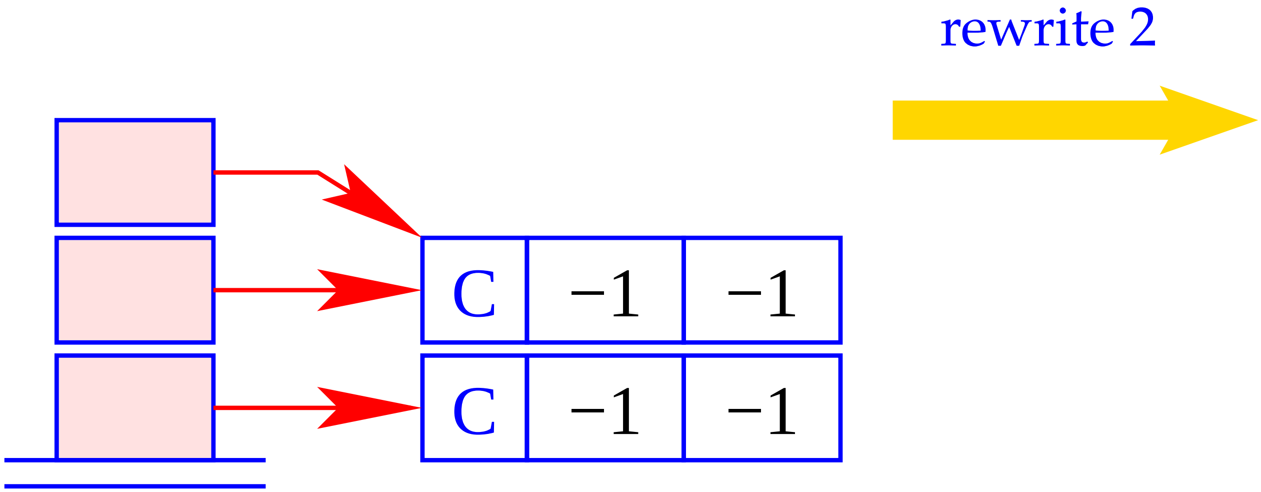
The execution of this instruction sequence should deliver the basic value 7 ...

```
0    alloc 2
2    pushloc 0
3    rewrite 2
2    loadc 7
3    mkbasic
3    rewrite 1
2    pushloc 1
3    eval
3    slide 2
```

```
0    alloc 2       3    rewrite 2     3    mkbasic       2    pushloc 1
2    pushloc 0     2    loadc 7       3    rewrite 1     3    eval
                                                         3    slide 2
```

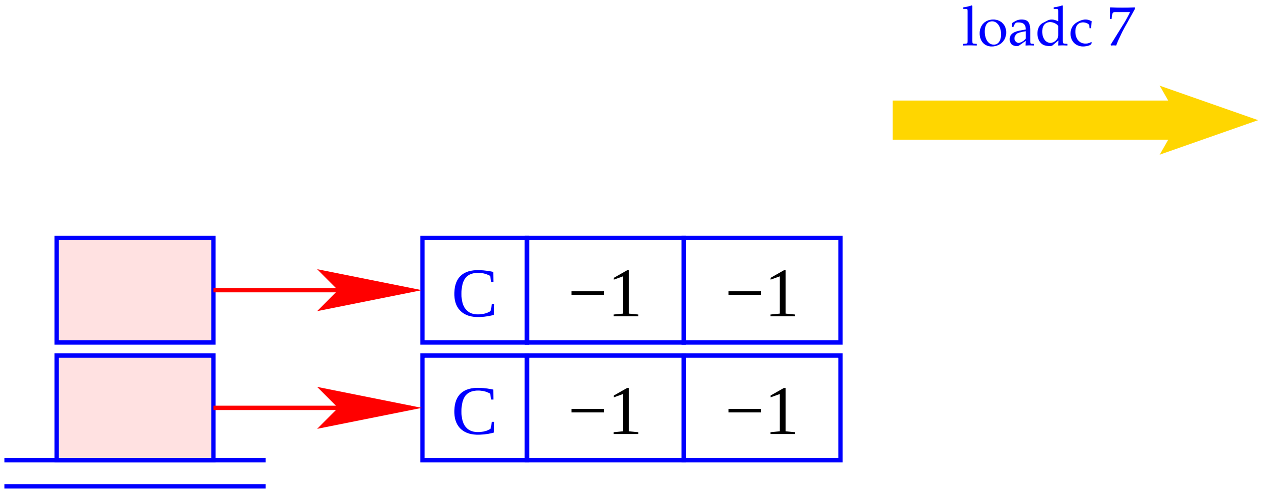


```
0    alloc 2        3    rewrite 2      3    mkbasic        2    pushloc 1
2    pushloc 0      2    loadc 7        3    rewrite 1      3    eval
                                                            3    slide 2
```

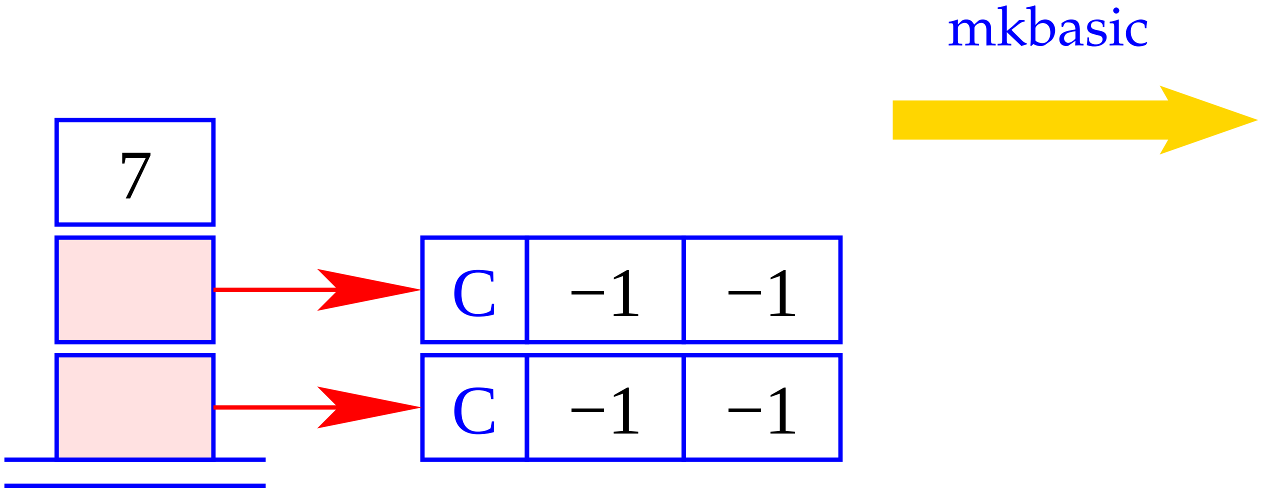


rewrite 2

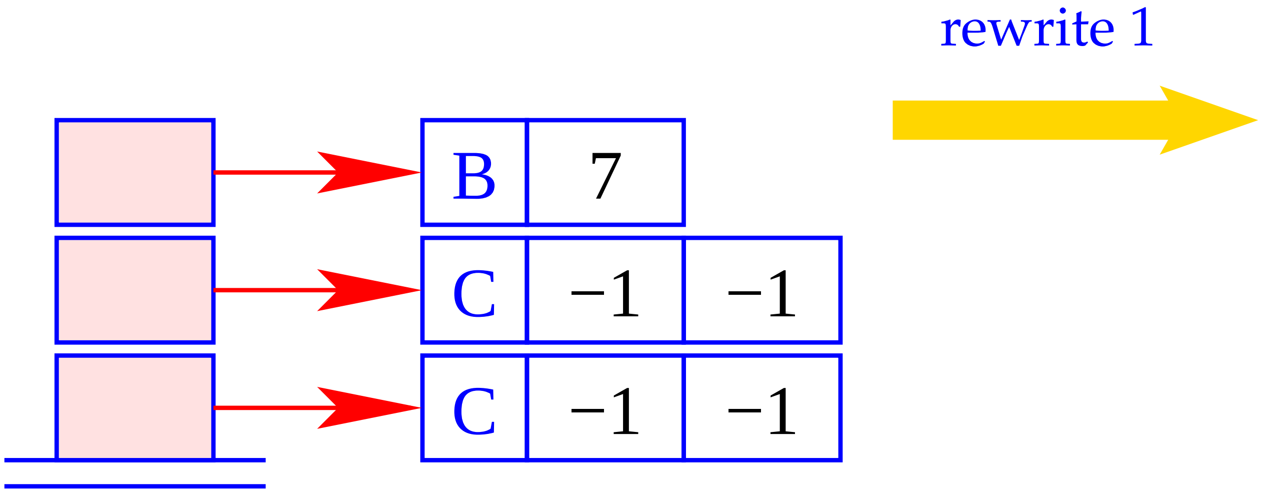
```
0   alloc 2
2   pushloc 0
3   rewrite 2
2   loadc 7
3   mkbasic
3   rewrite 1
2   pushloc 1
3   eval
3   slide 2
```



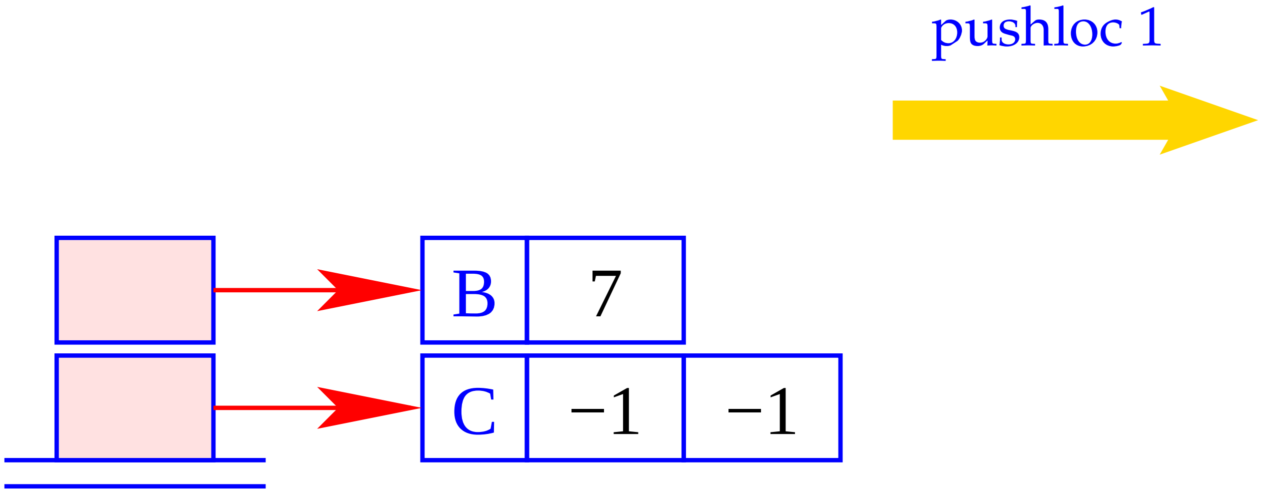
```
0   alloc 2
2   pushloc 0
3   rewrite 2
2   loadc 7
3   mkbasic
3   rewrite 1
2   pushloc 1
3   eval
3   slide 2
```



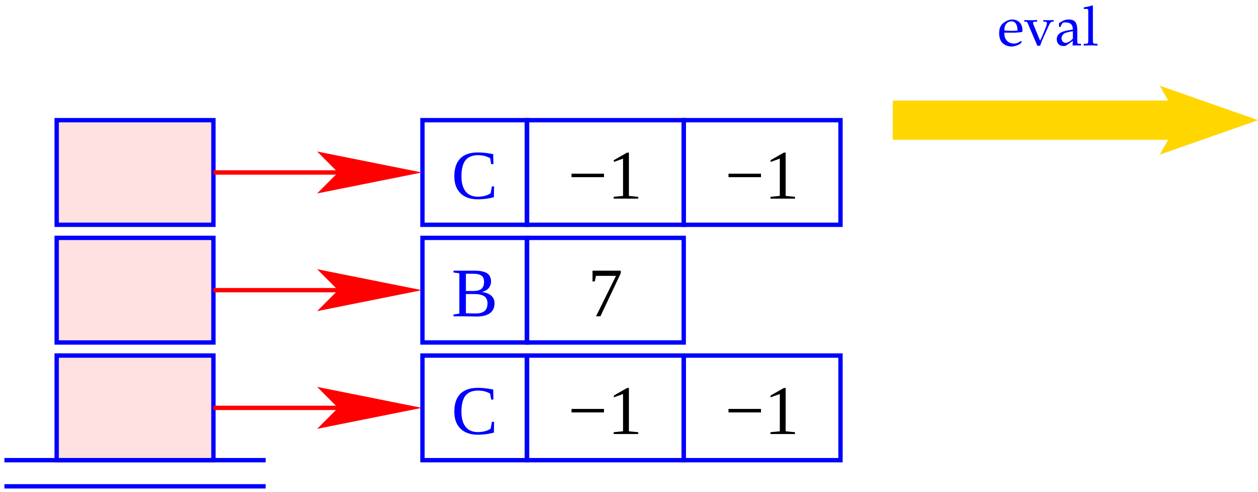
```
0    alloc 2
2    pushloc 0
3    rewrite 2
2    loadc 7
3    mkbasic
3    rewrite 1
2    pushloc 1
3    eval
3    slide 2
```



```
0    alloc 2
2    pushloc 0
3    rewrite 2
2    loadc 7
3    mkbasic
3    rewrite 1
2    pushloc 1
3    eval
3    slide 2
```



```
0    alloc 2
2    pushloc 0
3    rewrite 2
2    loadc 7
3    mkbasic
3    rewrite 1
2    pushloc 1
3    eval
3    slide 2
```



```
0    alloc 2
2    pushloc 0
3    rewrite 2
2    loadc 7
3    mkbasic
3    rewrite 1
2    pushloc 1
3    eval
3    slide 2
```

Segmentation Fault !!

Apparently, this optimization was not quite correct :-(

The Problem:

Binding of variable y to variable x before x's dummy node is replaced!!

$$\Longrightarrow$$

The Solution:

cyclic definitions: reject sequences of definitions like

$\textbf{let}\ a = b; \ldots b = a\ \textbf{in}\ \ldots$.

acyclic definitions: order the definitions $y = x$ such that the dummy node for the right side of x is already overwritten.

Functions:

Functions are values, which are not evaluated further. Instead of generating code that constructs a closure for an F-object, we generate code that constructs the F-object directly.

Therefore:

$$\mathrm{code}_C\ (\mathbf{fn}\ x_0, \ldots, x_{k-1} \Rightarrow e)\ \rho\ \mathrm{sd} \quad = \quad \mathrm{code}_V\ (\mathbf{fn}\ x_0, \ldots, x_{k-1} \Rightarrow e)\ \rho\ \mathrm{sd}$$

23 The Translation of a Program Expression

Execution of a program e starts with

$$\mathrm{PC} = 0 \qquad \mathrm{SP} = \mathrm{FP} = \mathrm{GP} = -1$$

The expression e must not contain free variables.

The value of e should be determined and then a halt instruction should be executed.

$$\begin{array}{lll} \mathrm{code}\; e & = & \mathrm{code}_V\; e\; \emptyset\; 0 \\ & & \mathrm{halt} \end{array}$$

Remarks:

- The code schemata as defined so far produce Spaghetti code.
- Reason: Code for function bodies and closures placed directly behind the instructions mkfunval resp. mkclos with a jump over this code.
- Alternative: Place this code somewhere else, e.g. following the halt-instruction:

  Advantage: Elimination of the direct jumps following mkfunval and mkclos.

  Disadvantage: The code schemata are more complex as they would have to accumulate the code pieces in a Code-Dump.

$\Longrightarrow$

Solution:

Disentangle the Spaghetti code in a subsequent optimization phase :-)

Example: $\textbf{let}\ a = 17;\ f = \textbf{fn}\ b \Rightarrow a + b\ \textbf{in}\ f\ 42$

Disentanglement of the jumps produces:

```
0      loadc 17
1      mkbasic
1      pushloc 0
2      mkvec 1
2      mkfunval A
2      mark B
5      loadc 42
6      mkbasic
6      pushloc 4
7      eval
7      apply
3  B:  slide 2
1      halt
0  A:  targ 1
0      pushglob 0
1      eval
1      getbasic
1      pushloc 1
2      eval
2      getbasic
2      add
1      mkbasic
1      return 1
```

24 Structured Data

In the following, we extend our functional programming language by some datatypes.

24.1 Tuples

Constructors: $(.,\dots,.)$, k-ary with $k \geq 0$;

Destructors: $\#j$ for $j \in \mathbb{N}_0$ (Projections)

We extend the syntax of expressions correspondingly:

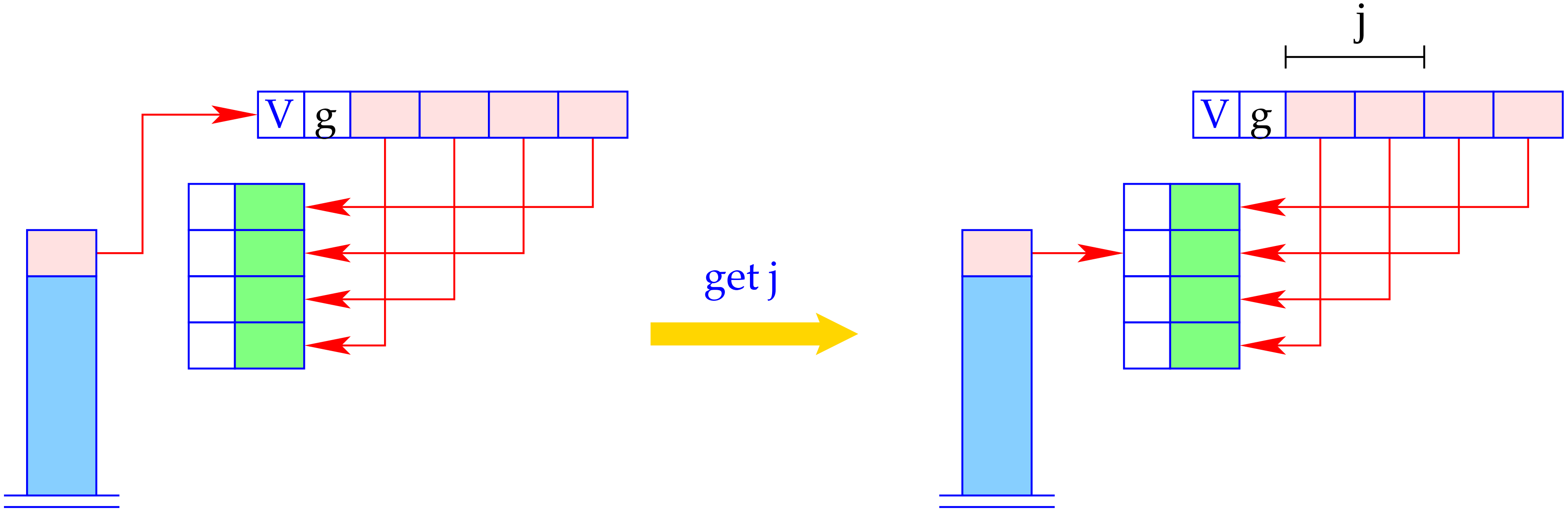
$$\begin{array}{lll} e & ::= & \dots \mid (e_0, \dots, e_{k-1}) \mid \#j\; e \\ & & \mid \;\textbf{let}\; (x_0, \dots, x_{k-1}) = e_1 \;\textbf{in}\; e_0 \end{array}$$

- In order to construct a tuple, we collect sequence of references on the stack. Then we construct a vector of these references in the heap using mkvec
- For returning components we use an indexed access into the tuple.

$$\begin{array}{lcl} \mathrm{code}_V\ (e_0, \ldots, e_{k-1})\ \rho\ \mathrm{sd} & = & \mathrm{code}_C\ e_0\ \rho\ \mathrm{sd} \\ & & \mathrm{code}_C\ e_1\ \rho\ (\mathrm{sd}+1) \\ & & \ldots \\ & & \mathrm{code}_C\ e_{k-1}\ \rho\ (\mathrm{sd}+k-1) \\ & & \mathrm{mkvec\ k} \end{array}$$

$$\begin{array}{lcl} \mathrm{code}_V\ (\#j\ e)\ \rho\ \mathrm{sd} & = & \mathrm{code}_V\ e\ \rho\ \mathrm{sd} \\ & & \mathrm{get\ j} \end{array}$$

In the case of CBV, we directly compute the values of the e_i.



```
if (S[SP] == (V,g,v))
   S[SP] = v[j];
else Error “Vector expected!”;
```

Inversion: Accessing all components of a tuple simulataneously:

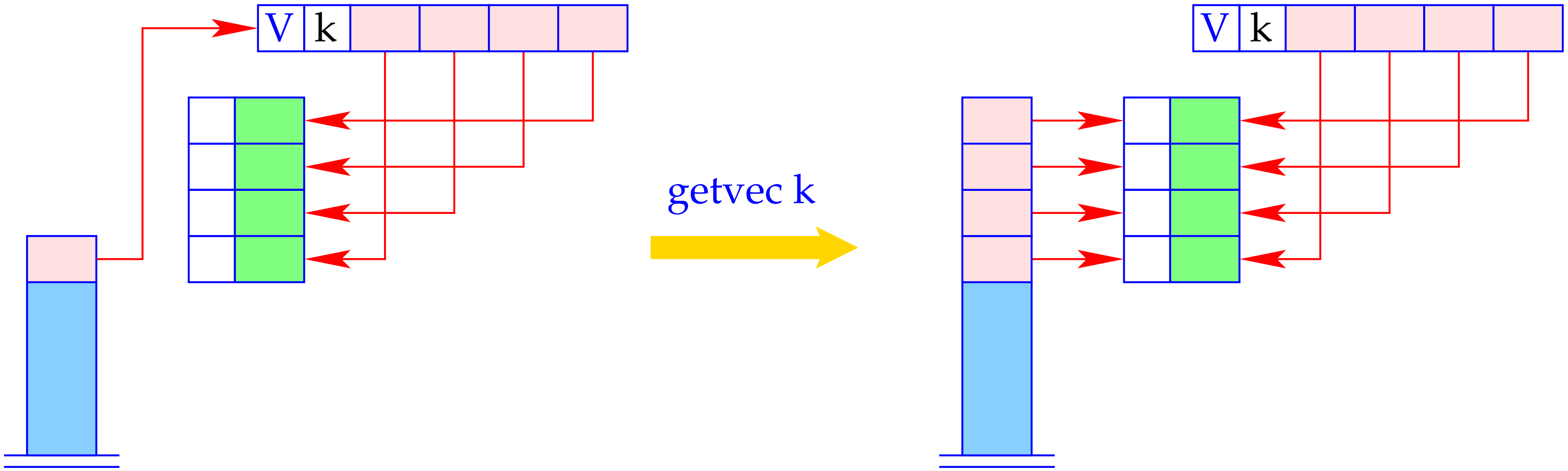
$$e \equiv \mathbf{let}\ (y_0, \ldots, y_{k-1}) = e_1\ \mathbf{in}\ e_0$$

This is translated as follows:

$$\begin{array}{lll} \mathrm{code}_V\ e\ \rho\ \mathrm{sd} & = & \mathrm{code}_V\ e_1\ \rho\ \mathrm{sd} \\ & & \mathrm{getvec\ k} \\ & & \mathrm{code}_V\ e_0\ \rho'\ (\mathrm{sd} + k) \\ & & \mathrm{slide\ k} \end{array}$$

where $\rho' = \rho \oplus \{y_i \mapsto (L, sd + i) \mid i = 0, \ldots, k-1\}$.

The instruction getvec k pushes the components of a vector of length k onto the stack:



```
if (S[SP] == (V,k,v)) {
   SP--;
   for(i=0; i<k; i++) {
         SP++; S[SP] = v[i];
         }
} else Error "Vector expected!";
```

24.2 Lists

Lists are constructed by the constructors:

[ ] “Nil”, the empty list;

“:” “Cons”, right-associative, takes an element and a list.

Access to list components is possible by **case**-expressions ...

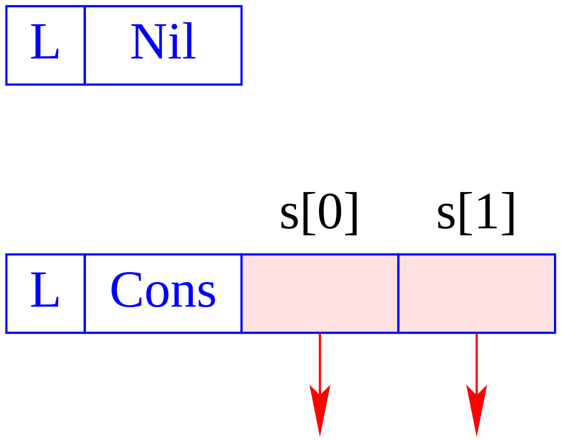
Example: The append function app:

$$
\begin{array}{lcl}
\text{app} & = & \textbf{fn}\ l, y \Rightarrow \textbf{case}\ l\ \textbf{of} \\
 & & \qquad \begin{array}{lcl} [] & \rightarrow & y \\ h : t & \rightarrow & h : (\text{app}\ t\ y) \end{array}
\end{array}
$$

accordingly, we extend the syntax of expressions:

$$\begin{array}{rcl} e & ::= & \ldots \mid [\,] \mid (e_1 : e_2) \\ & & \mid\ (\textbf{case}\ e_0\ \textbf{of}\ [\,] \to e_1;\ \ h : t \to e_2) \end{array}$$

Additionally, we need new heap objects:



L Nil

empty list

s[0] s[1]

L Cons

non−empty list

24.3 Building Lists

The new instructions nil and cons are introduced for building list nodes.

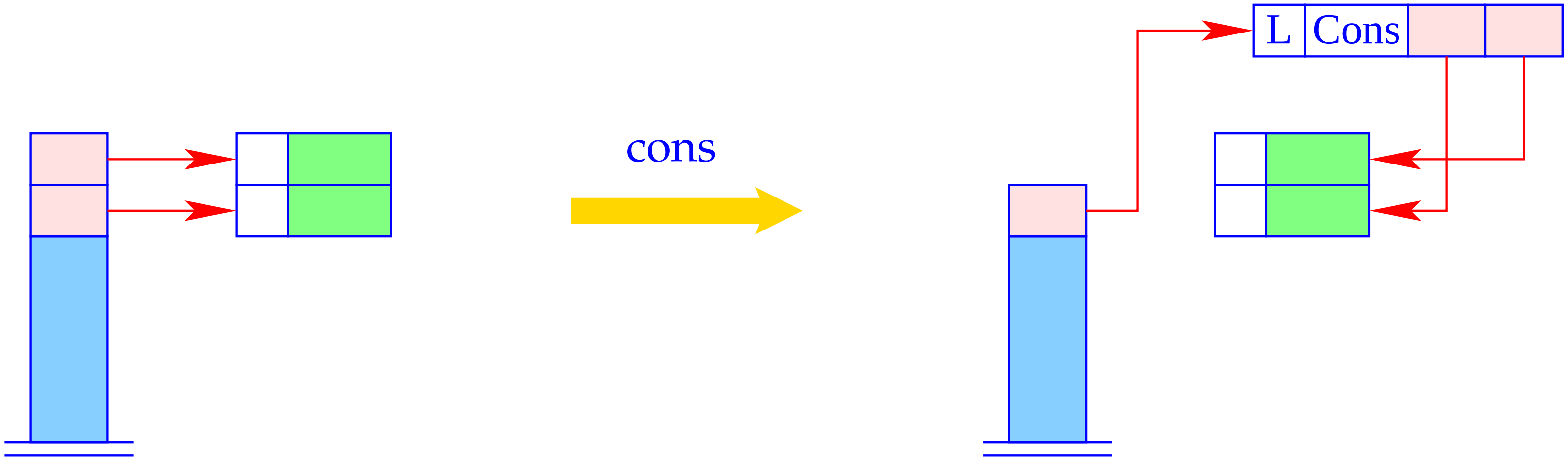
We translate for CBN:

$$
\begin{array}{lcl}
\text{code}_V\ []\ \rho\ \text{sd} & = & \text{nil} \\
\text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\
 & & \text{code}_C\ e_2\ \rho\ (\text{sd}+1) \\
 & & \text{cons}
\end{array}
$$

Note:

- With CBN: Closures are constructed for the arguments of “:”;
- With CBV: Arguments of “:” are evaluated :-)

S[SP] = SP++; S[SP] = new (L,Nil);



```
S[SP-1] = new (L,Cons, S[SP-1], S[SP]);
SP- -;
```

24.4 Pattern Matching

Consider the expression $e \equiv \mathbf{case}\ e_0\ \mathbf{of}\ [\,] \rightarrow e_1;\ \ h : t \rightarrow e_2.$

Evaluation of e requires:

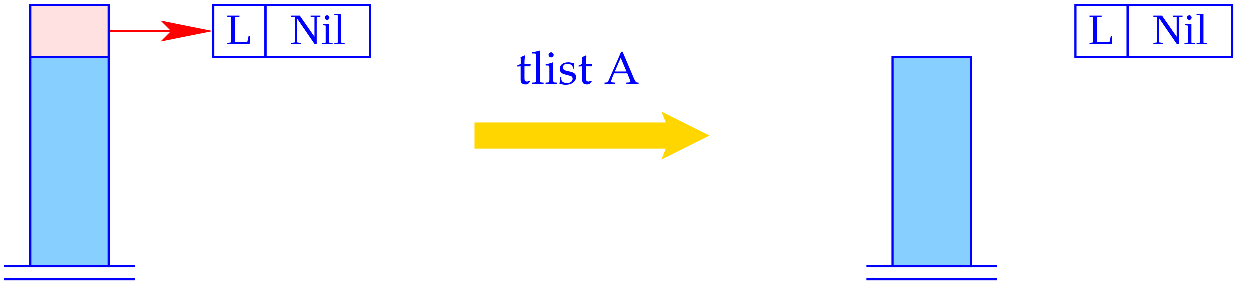
- evaluation of e_0;
- check, whether resulting value v is an L-object;
- if v is the empty list, evaluation of e_1 ...
- otherwise storing the two references of v on the stack and evaluation of e_2. This corresponds to binding h and t to the two components of v.

In consequence, we obtain (for CBN as for CBV):

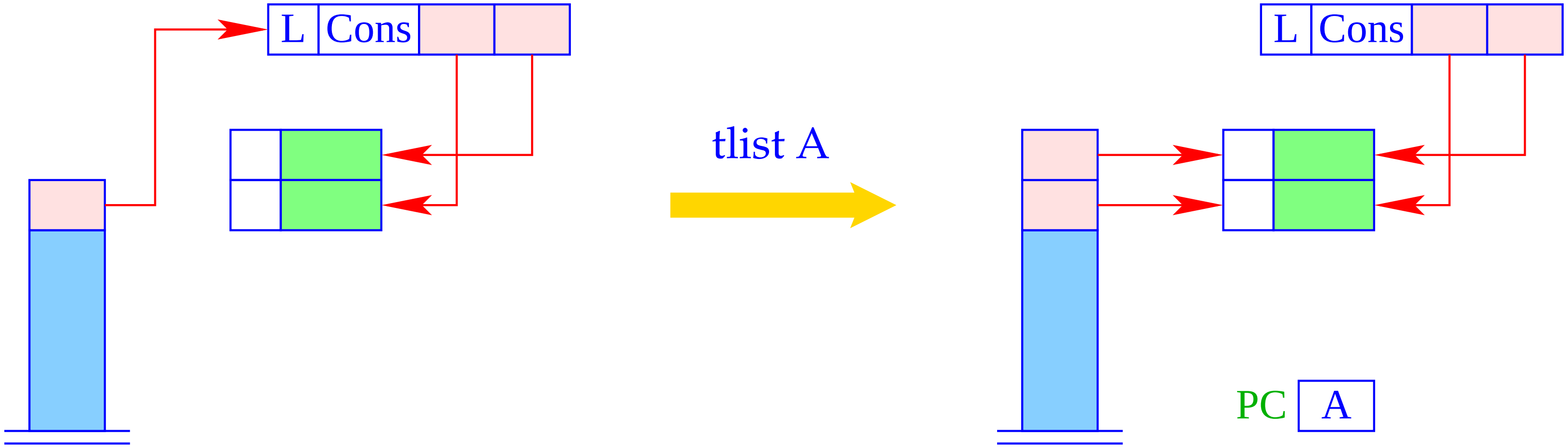
$$\begin{array}{lrl}
\mathrm{code}_V\; e\; \rho\; \mathrm{sd} & = & \mathrm{code}_V\; e_0\; \rho\; \mathrm{sd} \\
 & & \mathrm{tlist}\ \mathrm{A} \\
 & & \mathrm{code}_V\; e_1\; \rho\; \mathrm{sd} \\
 & & \mathrm{jump}\ \mathrm{B} \\
 & \mathrm{A}: & \mathrm{code}_V\; e_2\; \rho'\; (\mathrm{sd}+2) \\
 & & \mathrm{slide}\ 2 \\
 & \mathrm{B}: & \ldots
\end{array}$$

where $\rho' = \rho \oplus \{h \mapsto (L, sd+1), t \mapsto (L, sd+2)\}$.

The new instruction tlist A does the necessary checks and (in the case of Cons) allocates two new local variables:



```
h = S[SP];
if (H[h] != (L,...)
    Error “no list!”;
if (H[h] == (_,Nil)) SP- -;
      ...
```



```
... else {
   S[SP+1] = S[SP]→s[1];
   S[SP] = S[SP]→s[0];
   SP++; PC = A;
}
```

Example: The (disentangled) body of the function app with $\text{app} \mapsto (G, 0)$:

```
0          targ 2
0          pushloc 0
1          eval
1          tlist A
0          pushloc 1
1          eval
1          jump B
2    A:    pushloc 1
3          pushglob 0
4          pushloc 2
5          pushloc 6
6          mkvec 3
4          mkclos C
4          cons
0          slide 2
3    B:    return 2
0    C:    mark D
3          pushglob 2
4          pushglob 1
5          pushglob 0
6          eval
6          apply
1    D:    update
```

Note:

Datatypes with more than two constructors need a generalization of the tlist instruction, corresponding to a `switch`-instruction :-)

24.5 Closures of Tuples and Lists

The general schema for code_C can be optimized for tuples and lists:

$$
\begin{array}{lclcl}
\text{code}_C\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} & = & \text{code}_V\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} & = & \text{code}_C\ e_0\ \rho\ \text{sd} \\
 & & & & \text{code}_C\ e_1\ \rho\ (\text{sd} + 1) \\
 & & & & \ldots \\
 & & & & \text{code}_C\ e_{k-1}\ \rho\ (\text{sd} + k - 1) \\
 & & & & \text{mkvec k} \\
\text{code}_C\ []\ \rho\ \text{sd} & = & \text{code}_V\ []\ \rho\ \text{sd} & = & \text{nil} \\
\text{code}_C\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\
 & & & & \text{code}_C\ e_2\ \rho\ (\text{sd} + 1) \\
 & & & & \text{cons}
\end{array}
$$

25 Last Calls

A function application is called last call in an expression e if this application could deliver the value for e.

A last call usually is the outermost application of a defining expression.

A function definition is called tail recursive if all recursive calls are last calls.

Examples:

$r\ t\ (h : y)$ is a last call in $\quad$ $\textbf{case}\ x\ \textbf{of}\ [\,]\ \rightarrow y;\ \ h : t\ \rightarrow r\ t\ (h : y)$

$f\ (x - 1)$ is not a last call in $\quad$ $\textbf{if}\ x \leq 1\ \textbf{then}\ 1\ \textbf{else}\ x * f\ (x - 1)$

Observation: Last calls in a function body need no new stack frame!

$$\Longrightarrow$$

Automatic transformation of tail recursion into loops!!!

The code for a last call $l \equiv (e' \; e_0 \ldots e_{m_1})$ inside a function f with k arguments must

1. allocate the arguments e_i and evaluate e' to a function (note: all this inside f's frame!);
2. deallocate the local variables and the k consumed arguments of f;
3. execute an apply.

$$
\begin{array}{lcll}
\mathrm{code}_V \; l \; \rho \; \mathrm{sd} & = & \mathrm{code}_C \; e_{m-1} \; \rho \; \mathrm{sd} & \\
& & \mathrm{code}_C \; e_{m-2} \; \rho \; (\mathrm{sd}+1) & \\
& & \ldots & \\
& & \mathrm{code}_C \; e_0 \; \rho \; (\mathrm{sd}+m-1) & \\
& & \mathrm{code}_V \; e' \; \rho \; (\mathrm{sd}+m) & \text{// Evaluation of the function} \\
& & \mathrm{move} \; r \; (m+1) & \text{// Deallocation of } r \text{ cells} \\
& & \mathrm{apply} &
\end{array}
$$

where $r = sd + k$ is the number of stack cells to deallocate.

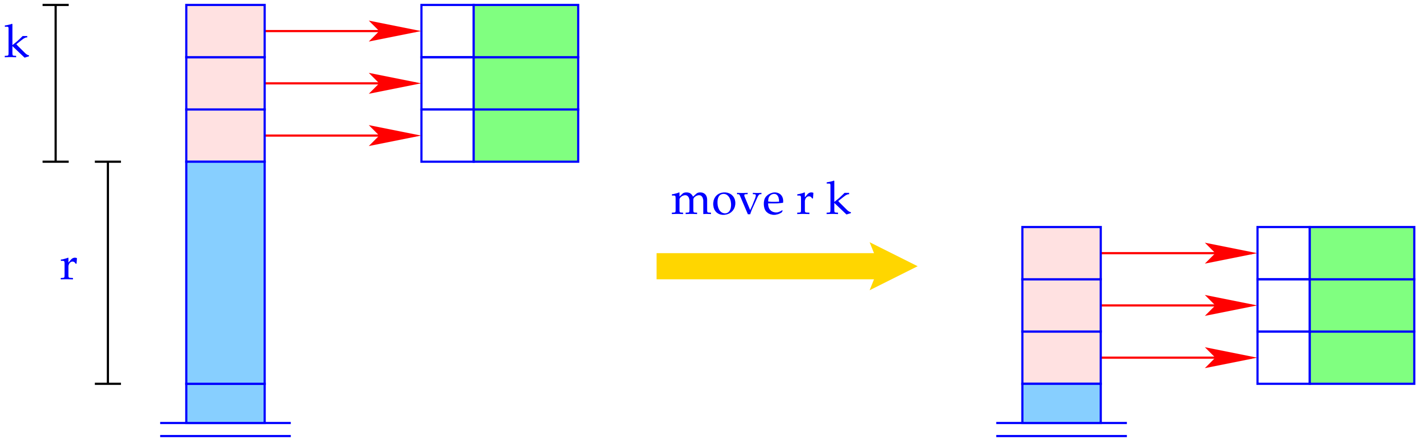
Example:

The body of the function

$$r = \textbf{fn}\, x, y \;\Rightarrow\; \textbf{case}\ x\ \textbf{of}\ [\,] \;\to y;\ h : t \;\to r\ t\ (h : y)$$

```
0    targ 2          1          jump B          4          pushglob 0
0    pushloc 0                                  5          eval
1    eval            2    A:    pushloc 1       5          move 4 3
1    tlist A         3          pushloc 4                  apply
0    pushloc 1       4          cons                       slide 2
1    eval            3          pushloc 1       1    B:    return 2
```

Since the old stack frame is kept, return 2 will only be reached by the direct jump at the end of the []-alternative.



```
SP = SP – k – r;
for (i=1; i≤k; i++)
    S[SP+i] = S[SP+i+r];
SP = SP + k;
```