24 Structured Data

In the following, we extend our functional programming language by some datatypes.

24.1 Tuples

Constructors: $(.,\ldots,.)$, k-ary with $k \geq 0$;

Destructors: $\#j$ for $j \in \mathbb{N}_0$ (Projections)

We extend the syntax of expressions correspondingly:

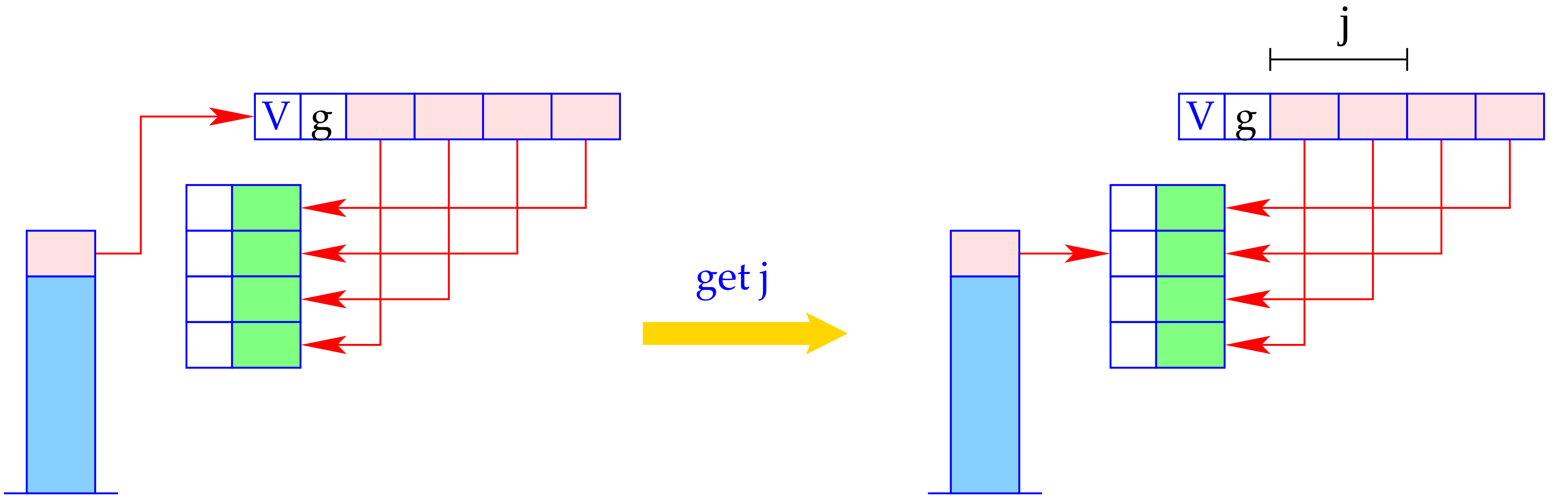
$$\begin{array}{lll} e & ::= & \ldots \mid (e_0,\ldots,e_{k-1}) \mid \#j\; e \\ & & \mid \;\textbf{let}\; (x_0,\ldots,x_{k-1}) = e_1 \;\textbf{in}\; e_0 \end{array}$$

- In order to construct a tuple, we collect sequence of references on the stack. Then we construct a vector of these references in the heap using mkvec
- For returning components we use an indexed access into the tuple.

$$\begin{array}{lll} \mathrm{code}_V\ (e_0, \ldots, e_{k-1})\ \rho\ \mathrm{sd} & = & \mathrm{code}_C\ e_0\ \rho\ \mathrm{sd} \\ & & \mathrm{code}_C\ e_1\ \rho\ (\mathrm{sd}+1) \\ & & \ldots \\ & & \mathrm{code}_C\ e_{k-1}\ \rho\ (\mathrm{sd}+k-1) \\ & & \mathrm{mkvec\ k} \end{array}$$

$$\begin{array}{lll} \mathrm{code}_V\ (\#j\ e)\ \rho\ \mathrm{sd} & = & \mathrm{code}_V\ e\ \rho\ \mathrm{sd} \\ & & \mathrm{get\ j} \\ & & \mathrm{eval} \end{array}$$

In the case of CBV, we directly compute the values of the e_i.



if (S[SP] == (V,g,v))
   S[SP] = v[j];
else Error “Vector expected!”;

Inversion: Accessing all components of a tuple simulataneously:

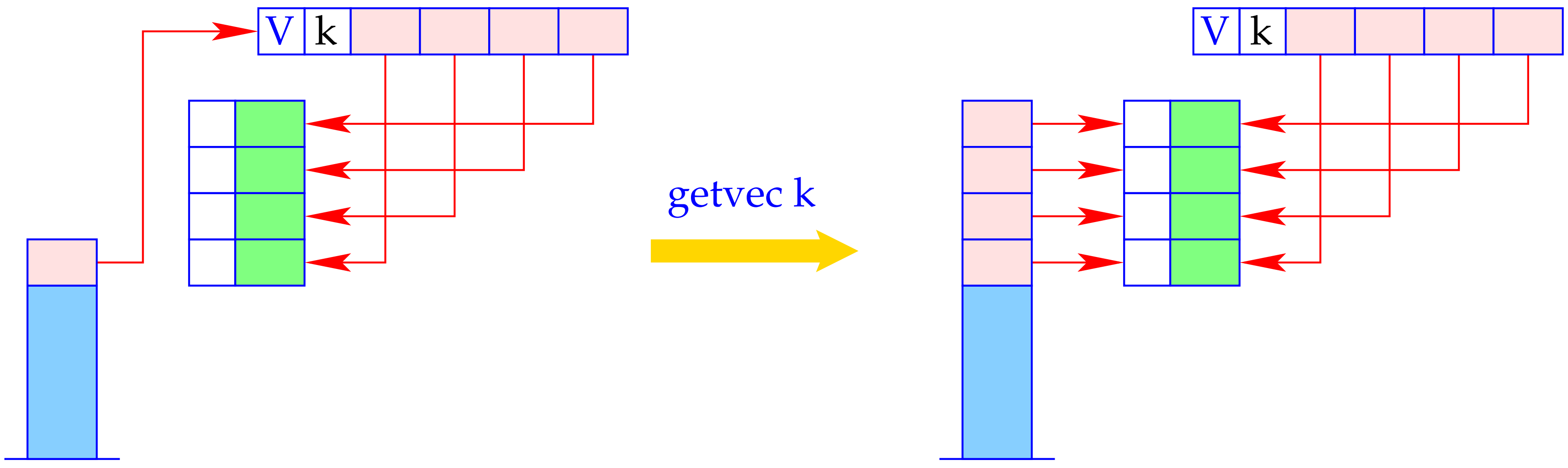
$$e \equiv \textbf{let } (y_0, \ldots, y_{k-1}) = e_1 \textbf{ in } e_0$$

This is translated as follows:

$$\begin{array}{lll} \mathrm{code}_V\; e\; \rho\; \mathrm{sd} & = & \mathrm{code}_V\; e_1\; \rho\; \mathrm{sd} \\ & & \mathrm{getvec\ k} \\ & & \mathrm{code}_V\; e_0\; \rho'\; (\mathrm{sd} + k) \\ & & \mathrm{slide\ k} \end{array}$$

where $\rho' = \rho \oplus \{y_i \mapsto (L, sd + i) \mid i = 0, \ldots, k-1\}$.

The instruction getvec k pushes the components of a vector of length k onto the stack:




```
if (S[SP] == (V,k,v)) {
   SP--;
   for(i=0; i<k; i++) {
        SP++; S[SP] = v[i];
        }
} else Error "Vector expected!";
```

24.2 Lists

Lists are constructed by the constructors:

[ ] "Nil", the empty list;

":" "Cons", right-associative, takes an element and a list.

Access to list components is possible by **case**-expressions ...

Example: The append function app:

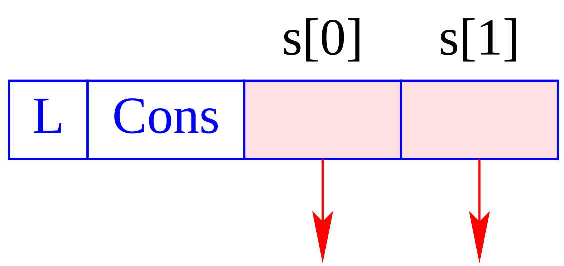
$$
\begin{array}{lll}
\text{app} & = & \textbf{fn}\ l, y \Rightarrow \textbf{case}\ l\ \textbf{of} \\
& & \qquad\qquad [\,] \quad \to \quad y \\
& & \qquad\qquad h : t \quad \to \quad h : (\text{app}\ t\ y)
\end{array}
$$

accordingly, we extend the syntax of expressions:

$$
\begin{array}{lll}
e & ::= & \ldots \mid [] \mid (e_1 : e_2) \\
 & & \mid (\textbf{case } e_0 \textbf{ of } [] \rightarrow e_1; \;\; h : t \rightarrow e_2)
\end{array}
$$

Additionally, we need new heap objects:

empty list



non−empty list

24.3 Building Lists

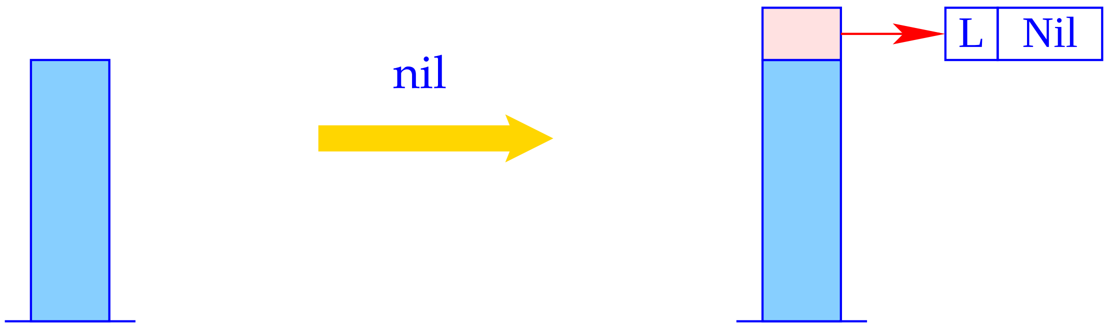
The new instructions nil and cons are introduced for building list nodes.

We translate for CBN:

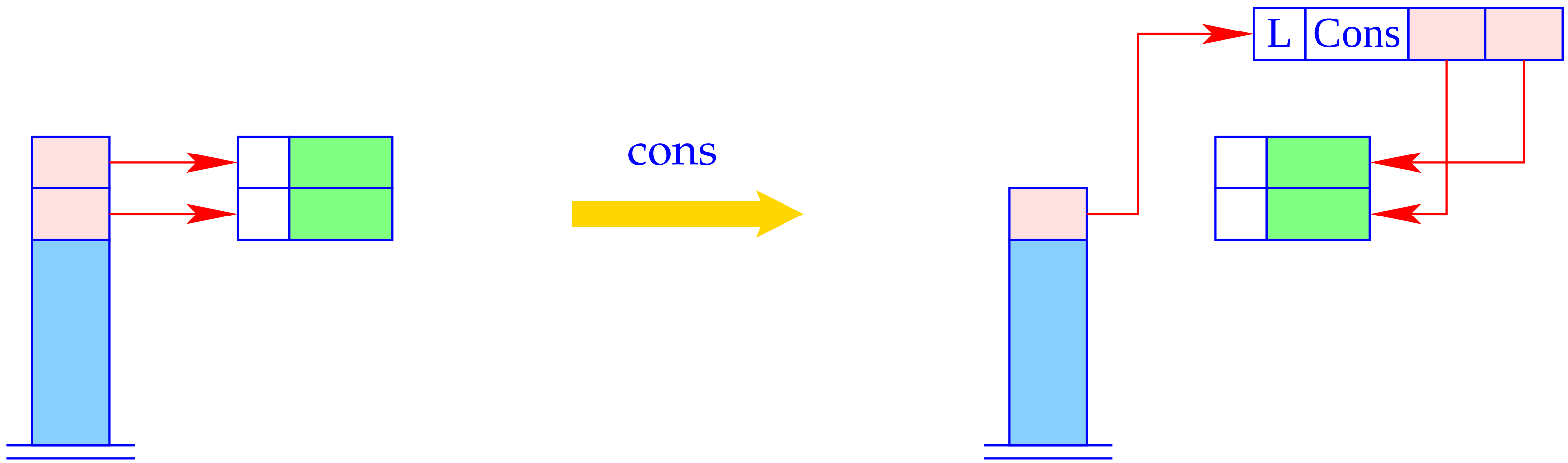
$$
\begin{array}{lll}
\text{code}_V\ []\ \rho\ \text{sd} & = & \text{nil} \\
\text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\
 & & \text{code}_C\ e_2\ \rho\ (\text{sd} + 1) \\
 & & \text{cons}
\end{array}
$$

Note:

- With CBN: Closures are constructed for the arguments of ":";
- With CBV: Arguments of ":" are evaluated :-)



S[SP] = SP++; S[SP] = new (L,Nil);



S[SP-1] = new (L,Cons, S[SP-1], S[SP]);
SP- -;

24.4 Pattern Matching

Consider the expression $e \equiv \textbf{case}\ e_0\ \textbf{of}\ [\,] \to e_1;\ \ h : t \to e_2.$

Evaluation of e requires:

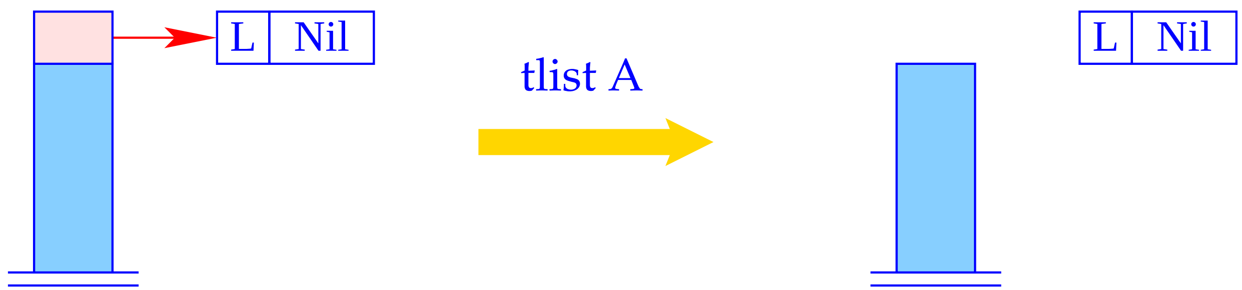
- evaluation of e_0;
- check, whether resulting value v is an L-object;
- if v is the empty list, evaluation of e_1 ...
- otherwise storing the two references of v on the stack and evaluation of e_2. This corresponds to binding h and t to the two components of v.

In consequence, we obtain (for CBN as for CBV):

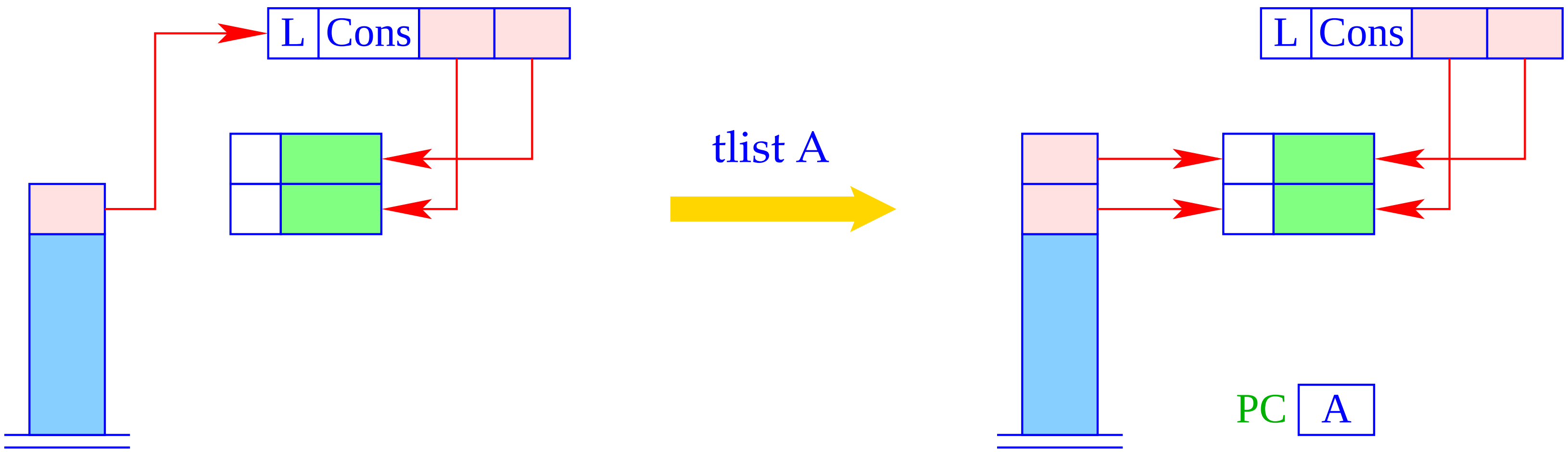
$$
\begin{array}{rrl}
\text{code}_V\ e\ \rho\ \text{sd} & = & \text{code}_V\ e_0\ \rho\ \text{sd} \\
 & & \text{tlist A} \\
 & & \text{code}_V\ e_1\ \rho\ \text{sd} \\
 & & \text{jump B} \\
 & \text{A}: & \text{code}_V\ e_2\ \rho'\ (\text{sd}+2) \\
 & & \text{slide 2} \\
 & \text{B}: & \ldots
\end{array}
$$

where $\rho' = \rho \oplus \{h \mapsto (L, sd+1), t \mapsto (L, sd+2)\}$.

The new instruction tlist A does the necessary checks and (in the case of Cons) allocates two new local variables:




```
h = S[SP];
if (H[h] != (L,...)
    Error “no list!”;
if (H[h] == (_,Nil)) SP- -;
      ...
```



... else {
   S[SP+1] = S[SP]→s[1];
   S[SP] = S[SP]→s[0];
   SP++; PC = A;
}

Example: The (disentangled) body of the function app with $\texttt{app} \mapsto (G, 0)$:

0		targ 2	3		pushglob 0	0	C:	mark D
0		pushloc 0	4		pushloc 2	3		pushglob 2
1		eval	5		pushloc 6	4		pushglob 1
1		tlist A	6		mkvec 3	5		pushglob 0
0		pushloc 1	4		mkclos C	6		eval
1		eval	4		cons	6		apply
1		jump B	0		slide 2	1	D:	update
2	A:	pushloc 1	3	B:	return 2			

Note:

Datatypes with more than two constructors need a generalization of the tlist instruction, corresponding to a `switch`-instruction :-)

24.5 Closures of Tuples and Lists

The general schema for code_C can be optimized for tuples and lists:

$$
\begin{array}{lclcl}
\text{code}_C\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} & = & \text{code}_V\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} & = & \text{code}_C\ e_0\ \rho\ \text{sd} \\
 & & & & \text{code}_C\ e_1\ \rho\ (\text{sd} + 1) \\
 & & & & \ldots \\
 & & & & \text{code}_C\ e_{k-1}\ \rho\ (\text{sd} + k - 1) \\
 & & & & \text{mkvec k} \\
\text{code}_C\ []\ \rho\ \text{sd} & = & \text{code}_V\ []\ \rho\ \text{sd} & = & \text{nil} \\
\text{code}_C\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_V\ (e_1 : e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\
 & & & & \text{code}_C\ e_2\ \rho\ (\text{sd} + 1) \\
 & & & & \text{cons}
\end{array}
$$

25 Last Calls

A function application is called last call in an expression e if this application could deliver the value for e.

A last call usually is the outermost application of a defining expression.

A function definition is called tail recursive if all recursive calls are last calls.

Examples:

$r\ t\ (h : y)$ is a last call in $\quad$ $\textbf{case}\ x\ \textbf{of}\ [\,]\ \to y;\ \ h : t\ \to r\ t\ (h : y)$

$f\ (x - 1)$ is not a last call in $\quad$ $\textbf{if}\ x \leq 1\ \textbf{then}\ 1\ \textbf{else}\ x * f\ (x - 1)$

Observation: Last calls in a function body need no new stack frame!

$$\Longrightarrow$$

Automatic transformation of tail recursion into loops!!!

The code for a last call $l \equiv (e' \; e_0 \ldots e_{m_1})$ inside a function f with k arguments must

1. allocate the arguments e_i and evaluate e' to a function (note: all this inside f's frame!);
2. deallocate the local variables and the k consumed arguments of f;
3. execute an apply.

$$
\begin{array}{lll}
\text{code}_V \; l \; \rho \; \text{sd} & = & \text{code}_C \; e_{m-1} \; \rho \; \text{sd} \\
 & & \text{code}_C \; e_{m-2} \; \rho \; (\text{sd} + 1) \\
 & & \ldots \\
 & & \text{code}_C \; e_0 \; \rho \; (\text{sd} + m - 1) \\
 & & \text{code}_V \; e' \; \rho \; (\text{sd} + m) \qquad\qquad // \text{ Evaluation of the function} \\
 & & \text{move} \; r \; (m+1) \qquad\qquad\quad // \text{ Deallocation of } r \text{ cells} \\
 & & \text{apply}
\end{array}
$$

where $r = sd + k$ is the number of stack cells to deallocate.

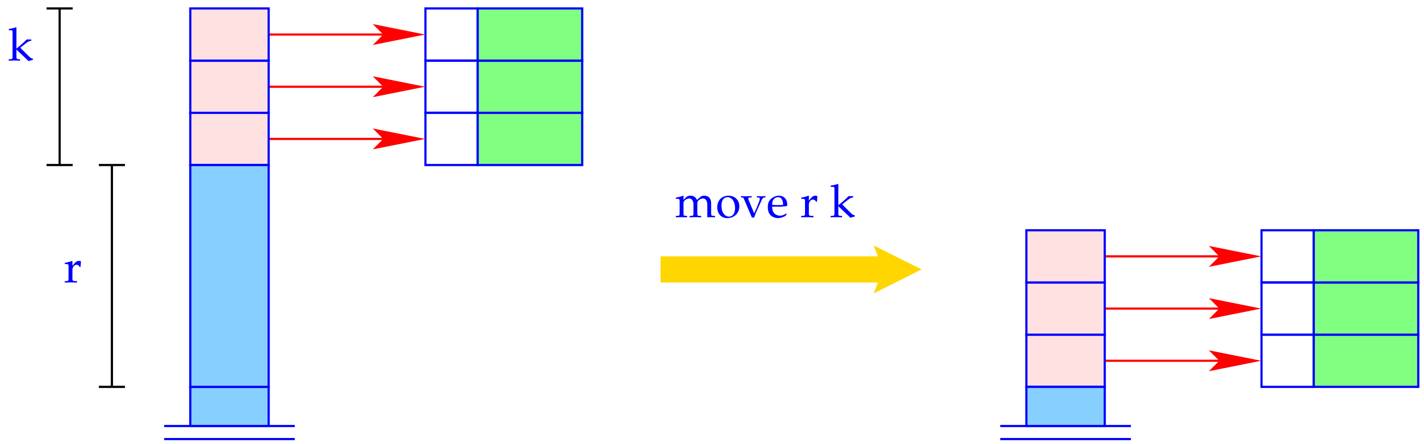
Example:

The body of the function

$$r = \textbf{fn}\, x, y \;\Rightarrow\; \textbf{case}\; x \;\textbf{of}\; [\,] \;\rightarrow y;\; h : t \;\rightarrow r\; t\; (h : y)$$

0		targ 2	1		jump B	4		pushglob 0
0		pushloc 0				5		eval
1		eval	2	A:	pushloc 1	5		move 4 3
1		tlist A	3		pushloc 4			apply
0		pushloc 1	4		cons			slide 2
1		eval	3		pushloc 1	1	B:	return 2

Since the old stack frame is kept, return 2 will only be reached by the direct jump at the end of the []-alternative.



SP = SP − k − r;
for (i=1; i≤k; i++)
    S[SP+i] = S[SP+i+r];
SP = SP + k;