# 19 letrec-Expressions

Consider the expression $\quad e \equiv \textbf{letrec}\ y_1 = e_1; \ldots; y_n = e_n\ \textbf{in}\ e_0 \quad .$

The translation of $e$ must deliver an instruction sequence that

- allocates local variables $y_1, \ldots, y_n$;
- in the case of
  CBV: evaluates $e_1, \ldots, e_n$ and binds the $y_i$ to their values;
  CBN: constructs closures for the $e_1, \ldots, e_n$ and binds the $y_i$ to them;
- evaluates the expression $e_0$ and returns its value.

## Warning:

In a **letrec**-expression, the definitions can use variables that will be allocated only later! $\Longrightarrow$ Dummy-values are put onto the stack before processing the definition.

For CBN, we obtain:

$$\begin{array}{rcll} \text{code}_V\; e\; \rho\; \text{sd} & = & \text{alloc n} & \text{// allocates local variables} \\ & & \text{code}_C\; e_1\; \rho'\; (\text{sd}+n) & \\ & & \text{rewrite n} & \\ & & \ldots & \\ & & \text{code}_C\; e_n\; \rho'\; (\text{sd}+n) & \\ & & \text{rewrite 1} & \\ & & \text{code}_V\; e_0\; \rho'\; (\text{sd}+n) & \\ & & \text{slide n} & \text{// deallocates local variables} \end{array}$$

where $\rho' = \rho \oplus \{y_i \mapsto (L, \text{sd}+i) \mid i = 1, \ldots, n\}$.

In the case of CBV, we also use $\text{code}_V$ for the expressions $e_1, \ldots, e_n$.

## Warning:

Recursive definitions of basic values are undefined with CBV!!!

## Example:

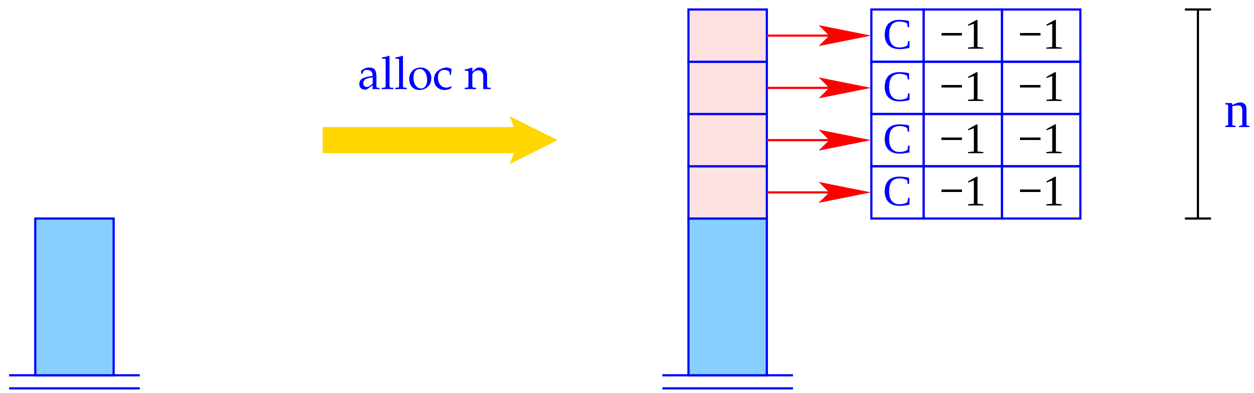
Consider the expression

$$e \equiv \textbf{letrec}\ f = \textbf{fn} x, y \Rightarrow \textbf{if} y \leq 1\ \textbf{then}\ x\ \textbf{else}\ f(x * y)(y-1)\ \textbf{in}\ f 1$$

for $\rho = \emptyset$ and sd $= 0$. We obtain (for CBV):

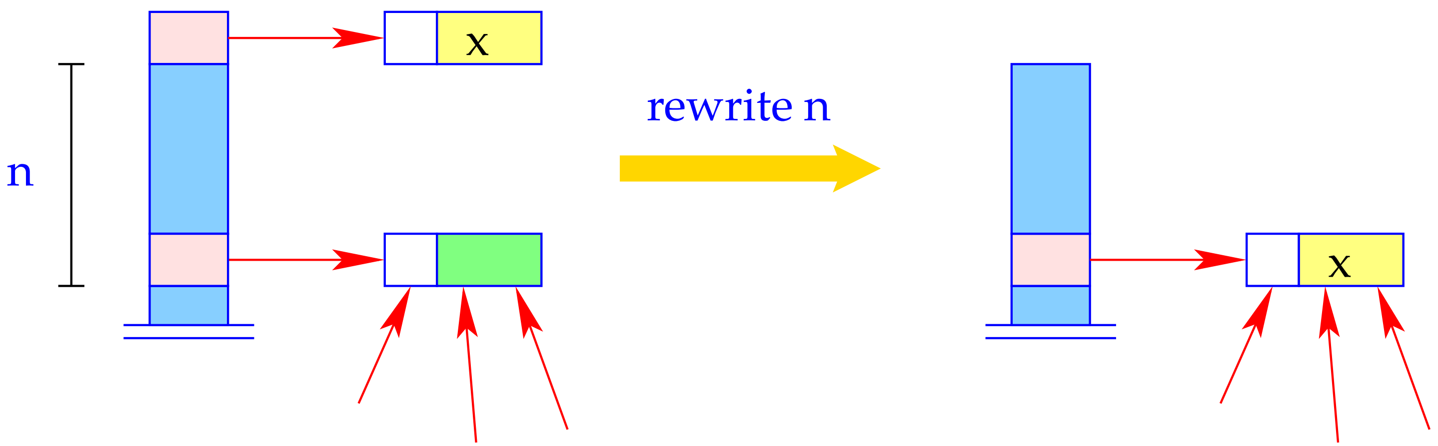
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | alloc 1 | 0 | A: | targ 2 | 4 | | loadc 1 |
| 1 | pushloc 0 | 0 | | ... | 5 | | mkbasic |
| 2 | mkvec 1 | 1 | | return 2 | 5 | | pushloc 4 |
| 2 | mkfunval A | 2 | B: | rewrite 1 | 6 | | apply |
| 2 | jump B | 1 | | mark C | 2 | C: | slide 1 |

The instruction alloc n reserves $n$ cells on the stack and initialises them with $n$ dummy nodes:




```
for (i=1; i<=n; i++)
    S[SP+i] = new (C,-1,-1);
SP = SP + n;
```

The instruction rewrite n overwrites the contents of the heap cell pointed to by the reference at S[SP–n]:




H[S[SP-n]] = H[S[SP]];
SP = SP - 1;

- The reference S[SP – n] remains unchanged!
- Only its contents is changed!

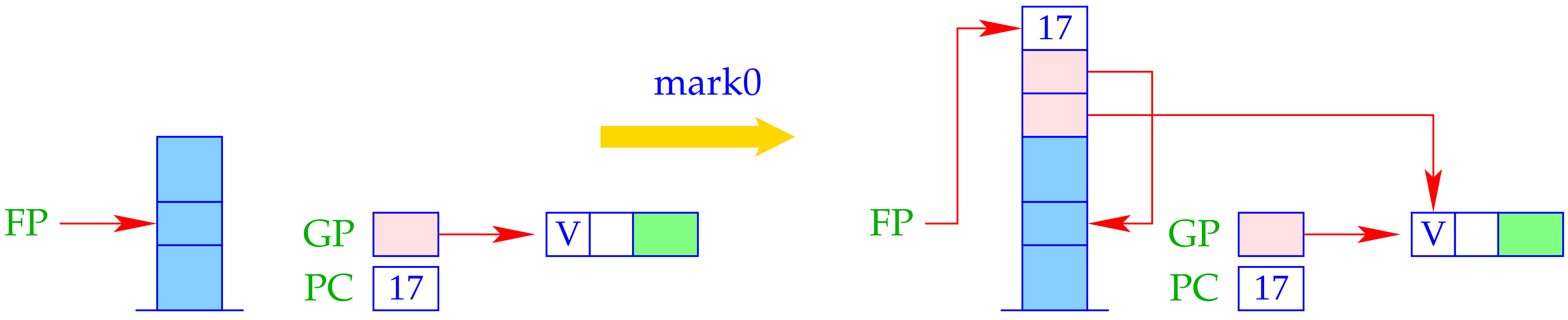
# 20 Closures and their Evaluation

- Closures are needed only for the implementation of CBN.
- Before the value of a variable is accessed (with CBN), this value must be available.
- Otherwise, a stack frame must be created to determine this value.
- This task is performed by the instruction eval.

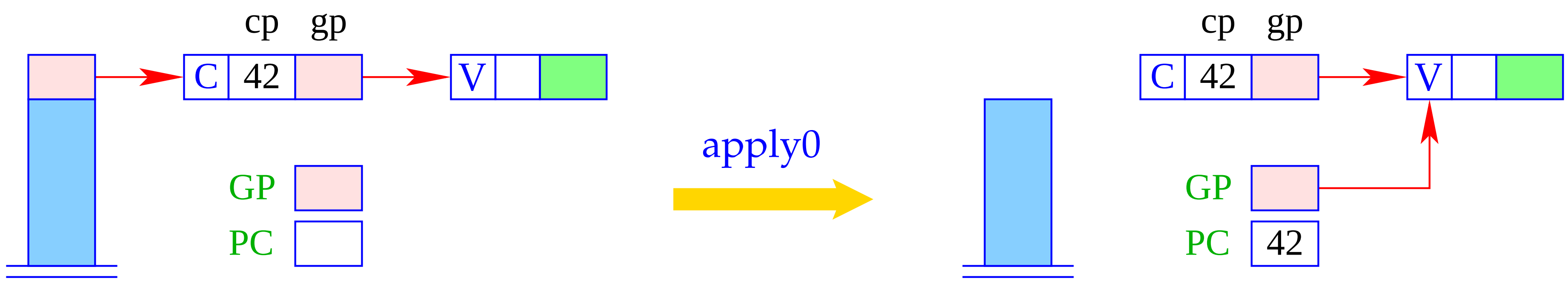
eval can be decomposed into small actions:

$$
\begin{aligned}
\text{eval} \quad = \quad & \text{if } (H[S[SP]] \equiv (C, \_, \_)) \; \{ \\
& \quad \text{mark0}; \qquad\qquad\qquad // \text{ allocation of the stack frame} \\
& \quad \text{pushloc } 3; \qquad\qquad // \text{ copying of the reference} \\
& \quad \text{apply0}; \qquad\qquad\quad // \text{ corresponds to apply} \\
& \}
\end{aligned}
$$

- A closure can be understood as a parameterless function. Thus, there is no need for an ap-component.
- Evaluation of the closure thus means evaluation of an application of this function to 0 arguments.
- In constrast to mark A , mark0 dumps the current PC.
- The difference between apply and apply0 is that no argument vector is put on the stack.

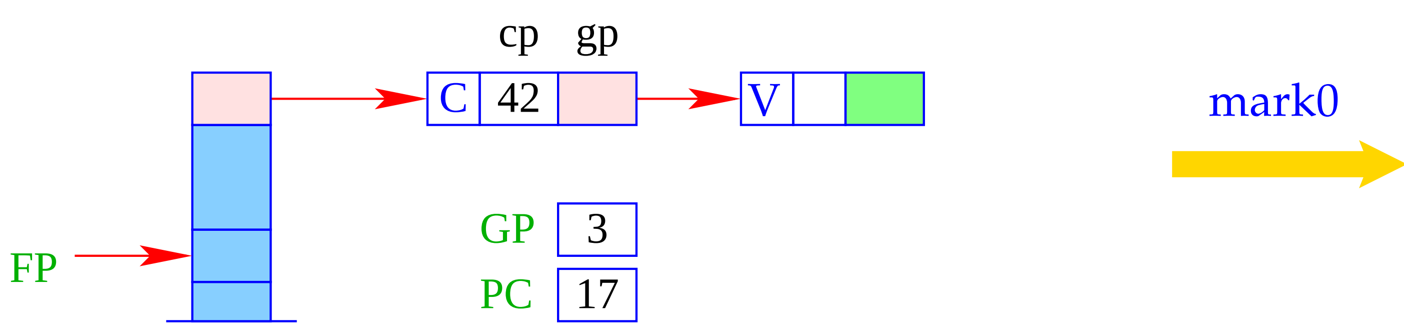


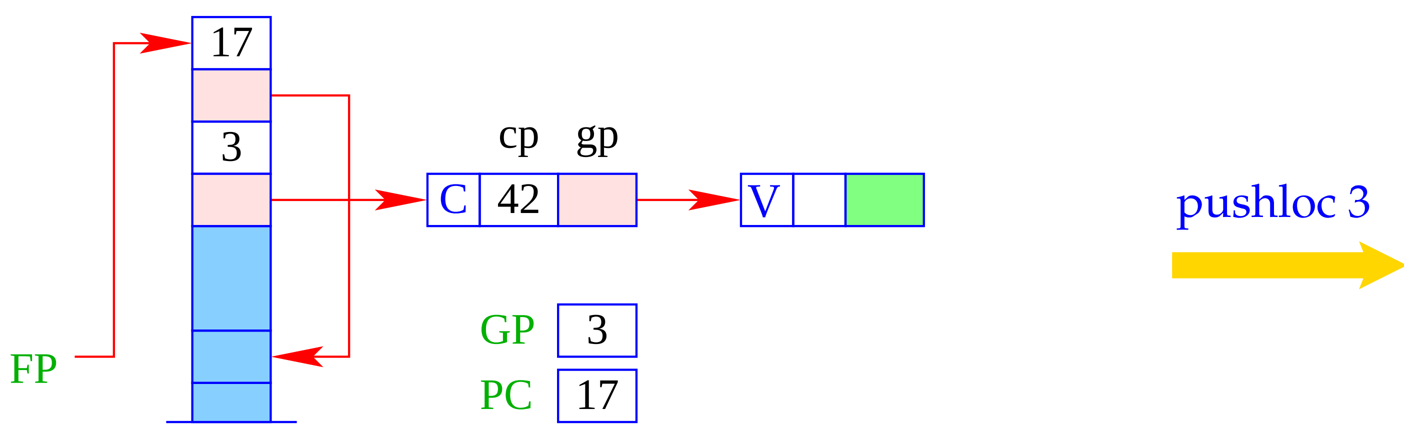

S[SP+1] = GP;
S[SP+2] = FP;
S[SP+3] = PC;
FP = SP = SP + 3;

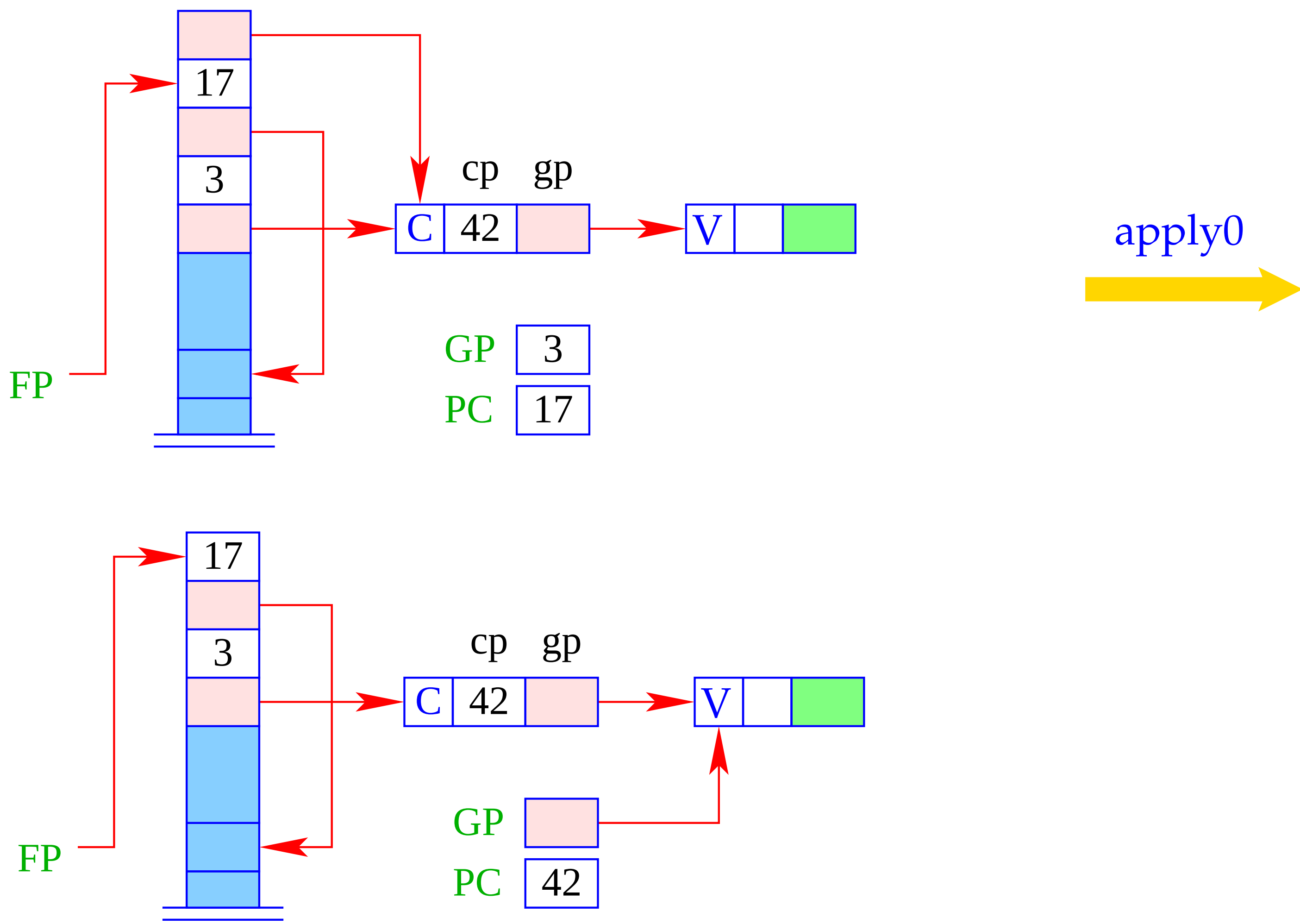



h = S[SP]; SP--;
GP = h→gp; PC = h→cp;

We thus obtain for the instruction eval:


cp gp
C 42
V
mark0
FP
GP 3
PC 17


17
3
cp gp
C 42
V
pushloc 3
FP
GP 3
PC 17


17
3
C
42
cp
gp
V
GP
3
PC
17
FP
apply0
17
3
C
42
cp
gp
V
GP
PC
42
FP

The construction of a closure for an expression $e$ consists of:

- Packing the bindings for the free variables into a vector;
- Creation of a C-object, which contains a reference to this vector and to the code for the evaluation of $e$:

$$
\begin{array}{rrl}
\text{code}_C\ e\ \rho\ \text{sd} & = & \text{getvar}\ z_0\ \rho\ \text{sd} \\
 & & \text{getvar}\ z_1\ \rho\ (\text{sd}+1) \\
 & & \ldots \\
 & & \text{getvar}\ z_{g-1}\ \rho\ (\text{sd}+g-1) \\
 & & \text{mkvec g} \\
 & & \text{mkclos A} \\
 & & \text{jump B} \\
 & \text{A}: & \text{code}_V\ e\ \rho'\ 0 \\
 & & \text{update} \\
 & \text{B}: & \ldots
\end{array}
$$

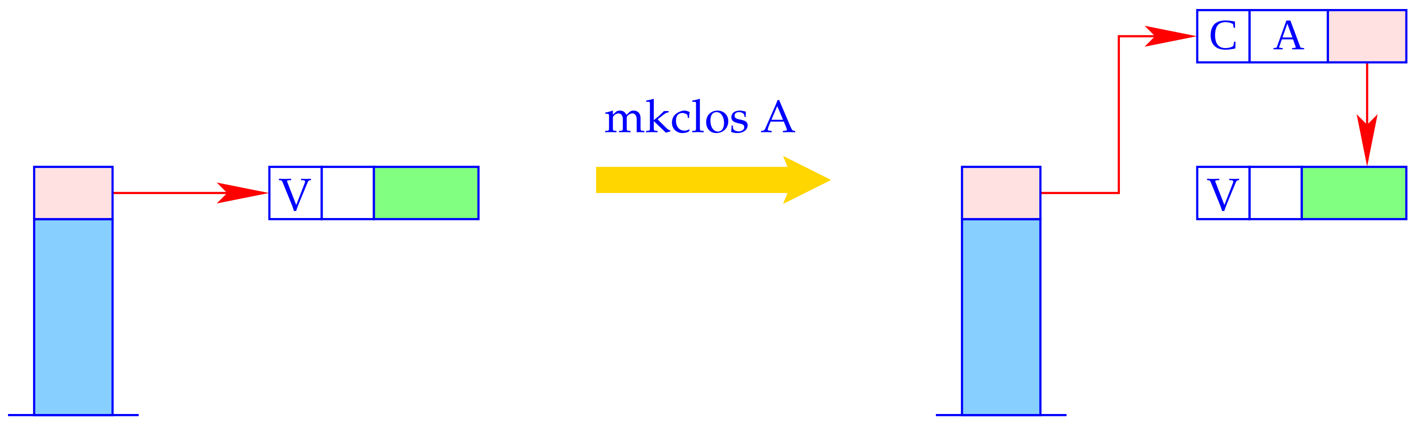
where $\{z_0, \ldots, z_{g-1}\} = \mathit{free}(e)$ and $\rho' = \{z_i \mapsto (G, i) \mid i = 0, \ldots, g-1\}$.

## Example:

Consider $e \equiv a * a$ with $\rho = \{a \mapsto (L, 0)\}$ and sd $= 1$. We obtain:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | pushloc 1 | 0 | A: | pushglob 0 | 2 | | getbasic |
| 2 | mkvec 1 | 1 | | eval | 2 | | mul |
| 2 | mkclos A | 1 | | getbasic | 1 | | mkbasic |
| 2 | jump B | 1 | | pushglob 0 | 1 | | update |
| | | 2 | | eval | 2 | B: | ... |

- The instruction mkclos A is analogous to the instruction mkfunval A.
- It generates a C-object, where the included code pointer is A.



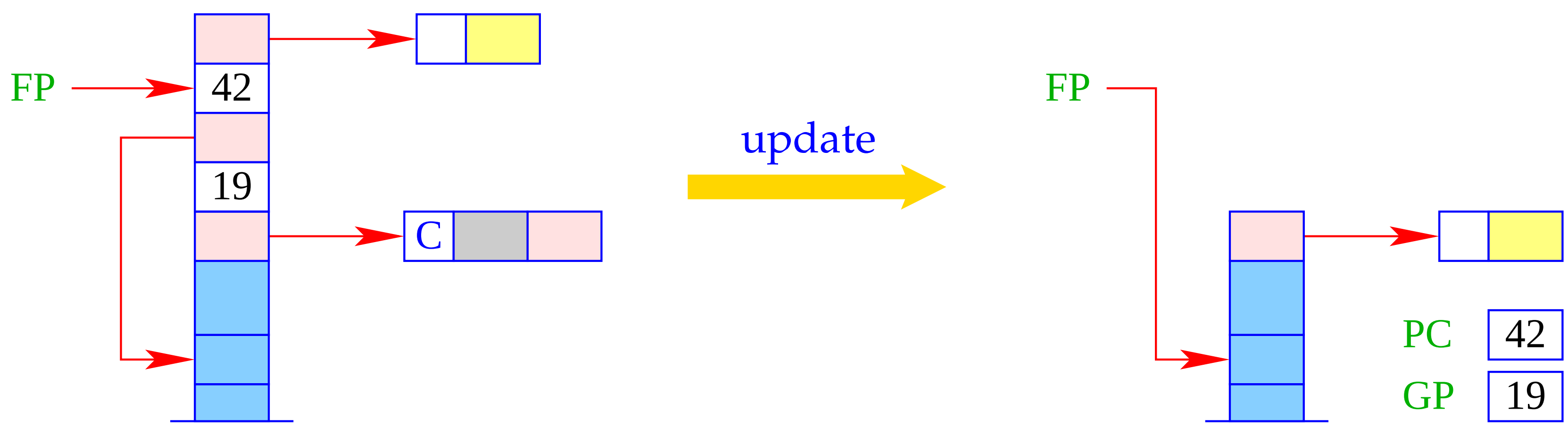

S[SP] = new (C, A, S[SP]);

In fact, the instruction update is the combination of the two actions:

popenv

rewrite 1

It overwrites the closure with the computed value.



# 21 Optimizations I: Global Variables

Observation:

- Functional programs construct many F- and C-objects.
- This requires the inclusion of (the bindings of) all global variables.

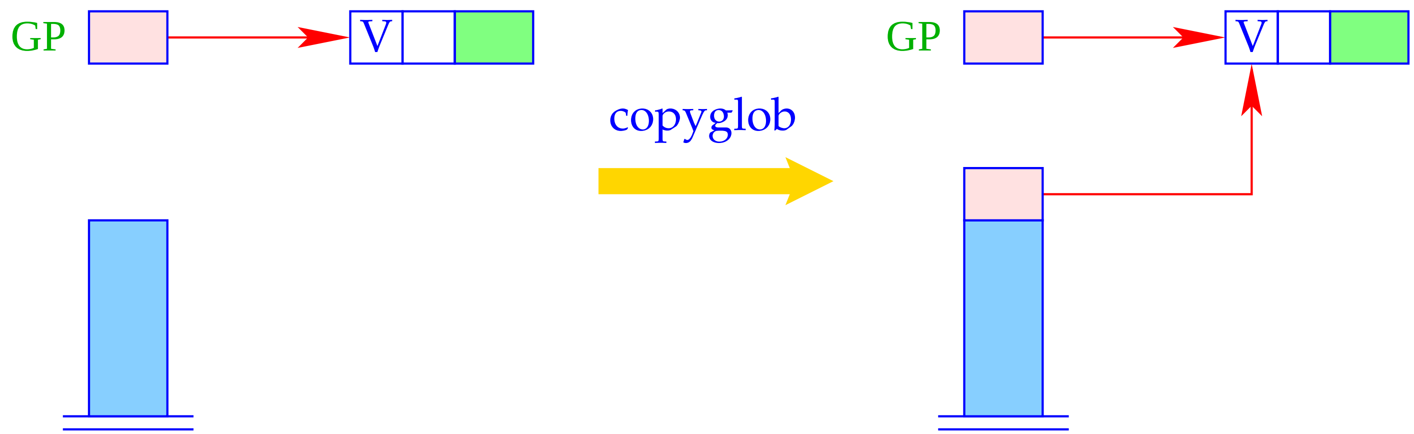
  Recall, e.g., the construction of a closure for an expression $e$ ...

$$
\begin{array}{rrl}
\text{code}_C\ e\ \rho\ \text{sd} & = & \text{getvar}\ z_0\ \rho\ \text{sd} \\
& & \text{getvar}\ z_1\ \rho\ (\text{sd}+1) \\
& & \ldots \\
& & \text{getvar}\ z_{g-1}\ \rho\ (\text{sd}+g-1) \\
& & \text{mkvec g} \\
& & \text{mkclos A} \\
& & \text{jump B} \\
& \text{A}: & \text{code}_V\ e\ \rho'\ 0 \\
& & \text{update} \\
& \text{B}: & \ldots
\end{array}
$$

where $\{z_0, \ldots, z_{g-1}\} = \mathit{free}(e)$ and $\rho' = \{z_i \mapsto (G, i) \mid i = 0, \ldots, g-1\}$.

Idea:

- Reuse Global Vectors, i.e. share Global Vectors!
- Profitable in the translation of **let**-expressions or function applications: Build one Global Vector for the union of the free-variable sets of all let-definitions resp. all arguments.
- Allocate (references to ) global vectors with multiple uses in the stack frame like local variables!
- Support the access to the current GP by an instruction copyglob :




SP++;
S[SP] = GP;

- The optimization will cause Global Vectors to contain more components than just references to the free the variables that occur in one expression ...

**Disadvantage:** Superfluous components in Global Vectors prevent the deallocation of already useless heap objects $\Longrightarrow$ Space Leaks :-(

**Potential Remedy:** Deletion of references at the end of their life time.

# 22 Optimizations II: Closures

In some cases, the construction of closures can be avoided, namely for

- Basic values,
- Variables,
- Functions.

## Basic Values:

The construction of a closure for the value is at least as expensive as the construction of the B-object itself!

Therefore:

$$\text{code}_C\ b\ \rho\ \text{sd} \quad = \quad \text{code}_V\ b\ \rho\ \text{sd} \quad = \quad \begin{array}{l}\text{loadc b}\\ \text{mkbasic}\end{array}$$

This replaces:

| | | | | | | |
|---|---|---|---|---|---|---|
| mkvec 0 | | jump B | mkbasic | B: | ... |
| mkclos A | A: | loadc b | update | | |

## Variables:

Variables are either bound to values or to C-objects. Constructing another closure is therefore superfluous. Therefore:

$$\text{code}_C\; x\; \rho\; \text{sd} \quad = \quad \text{getvar}\; x\; \rho\; \text{sd}$$

This replaces:

| | | | | | |
|---|---|---|---|---|---|
| getvar $x$ $\rho$ sd | mkclos A | A: | pushglob 0 | | update |
| mkvec 1 | jump B | | eval | B: | ... |

**Example:** $e \equiv \textbf{letrec}\; a = b; b = 7\; \textbf{in}\; a.$ $\quad \text{code}_V\; e\; \emptyset\; 0$ produces:

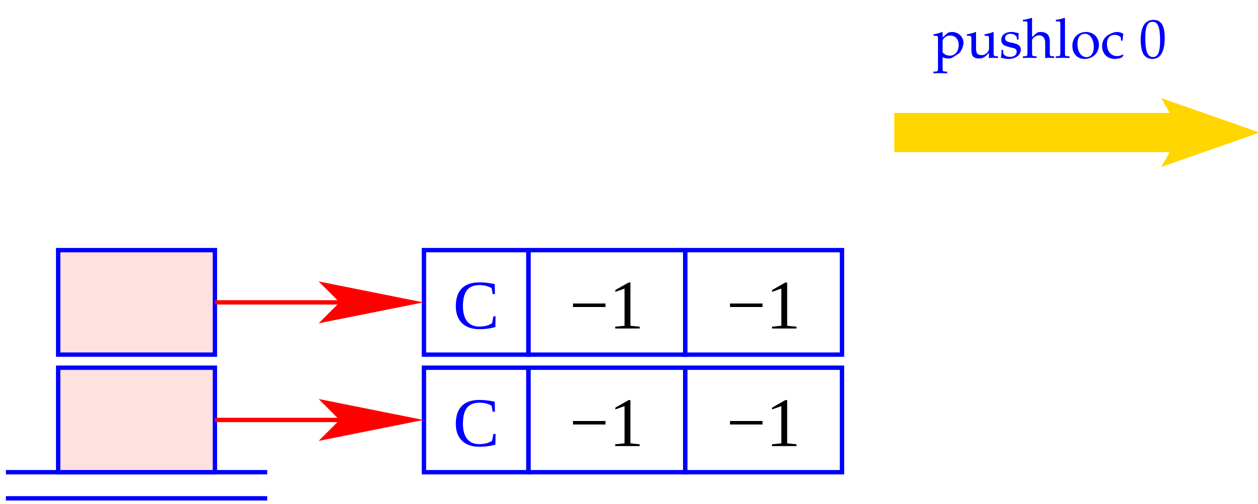
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |

The execution of this instruction sequence should deliver the basic value 7 ...

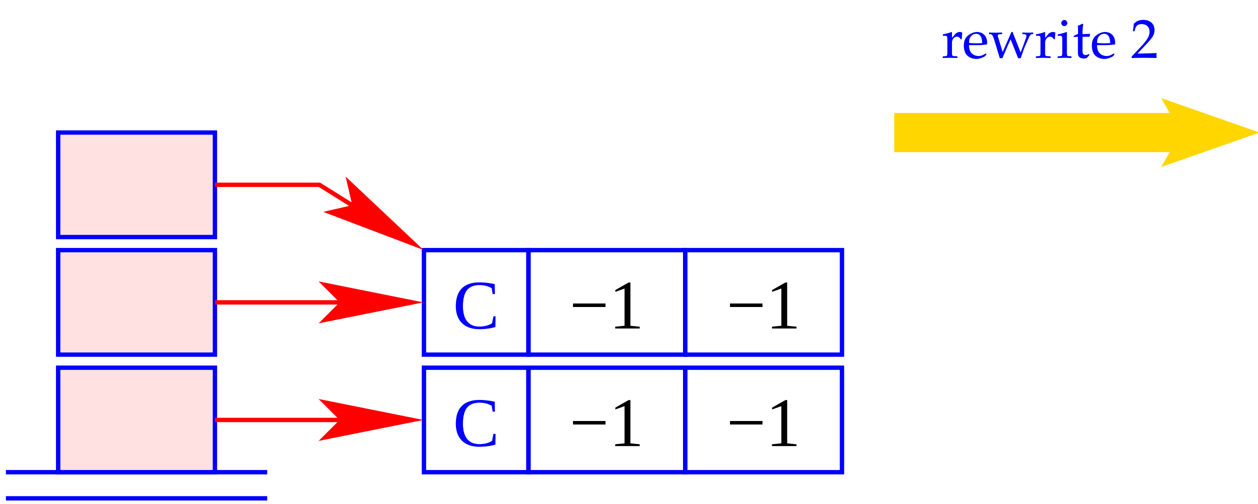
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |

alloc 2

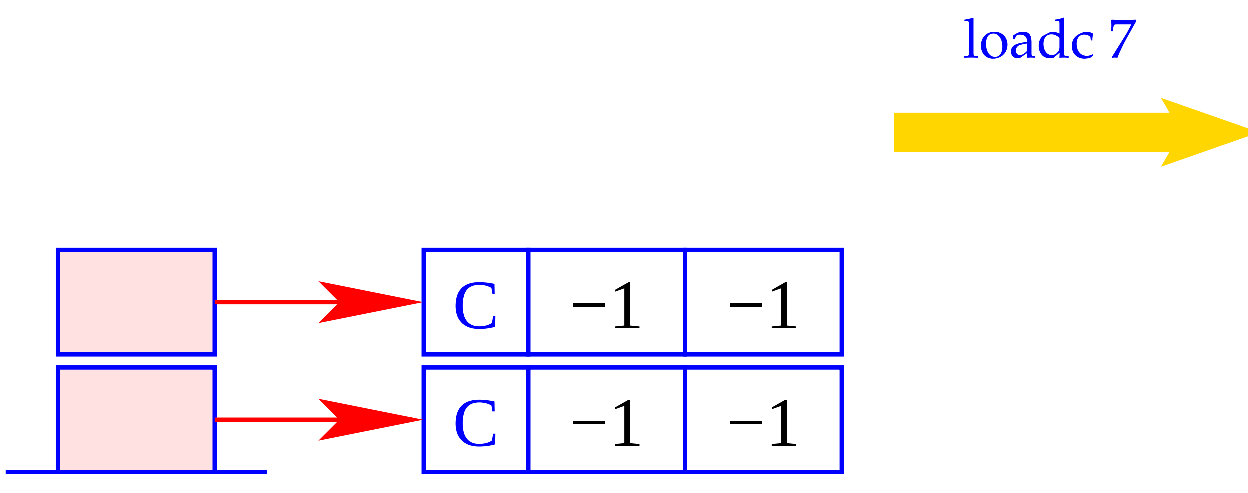
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



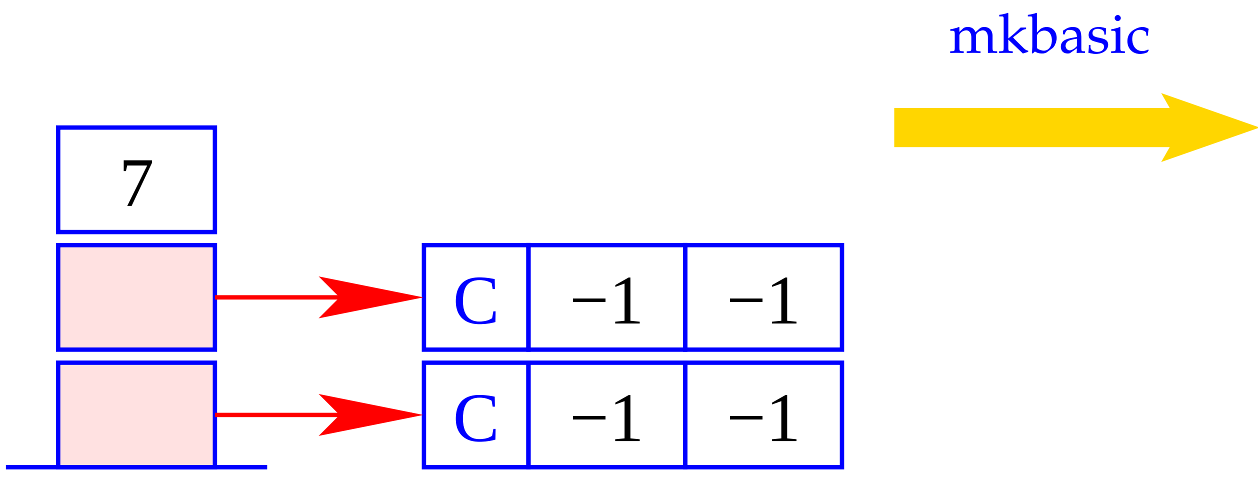
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



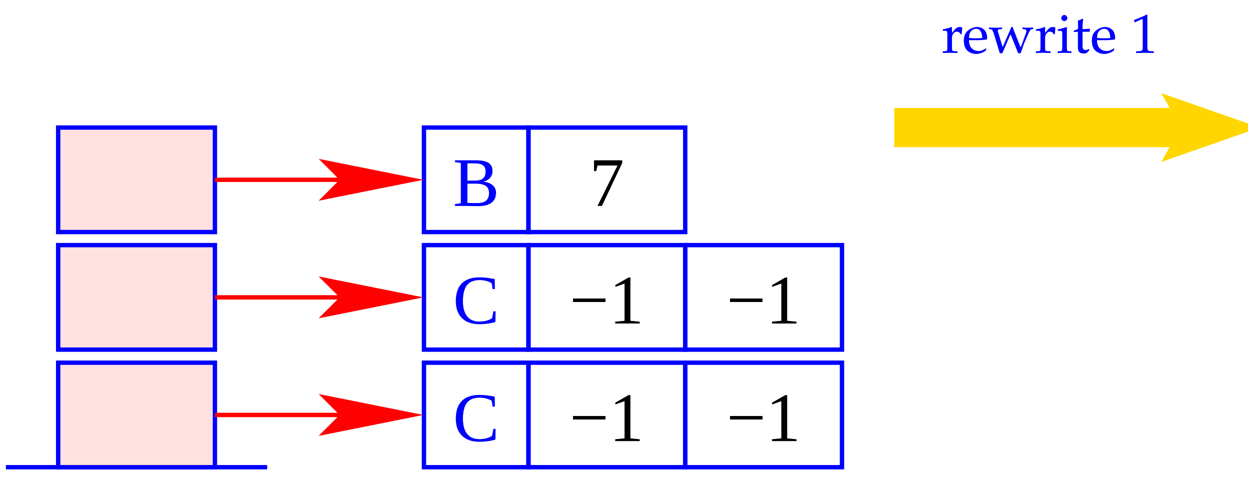
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



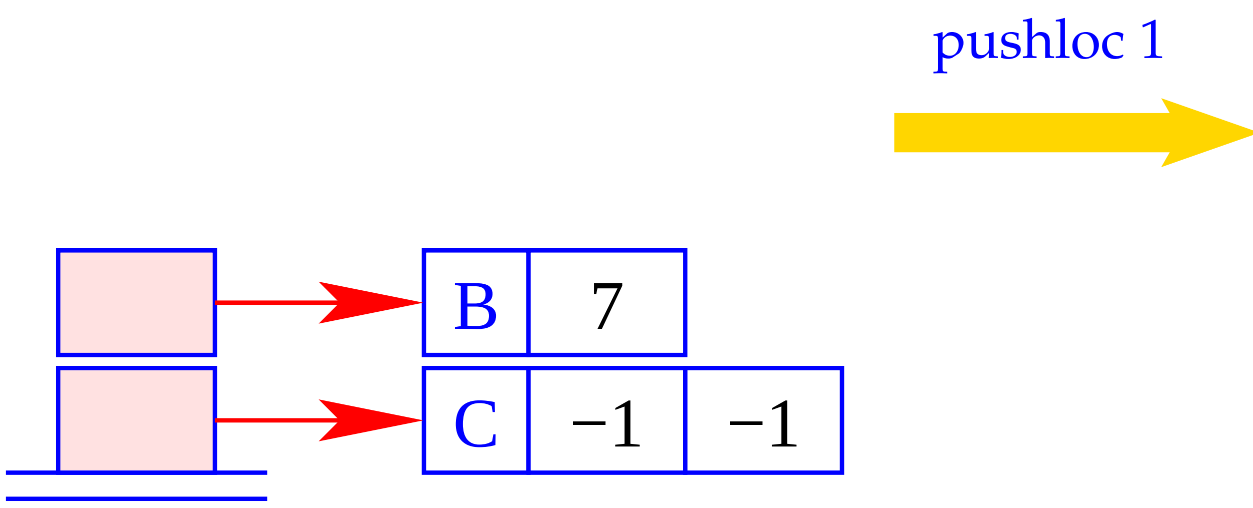
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



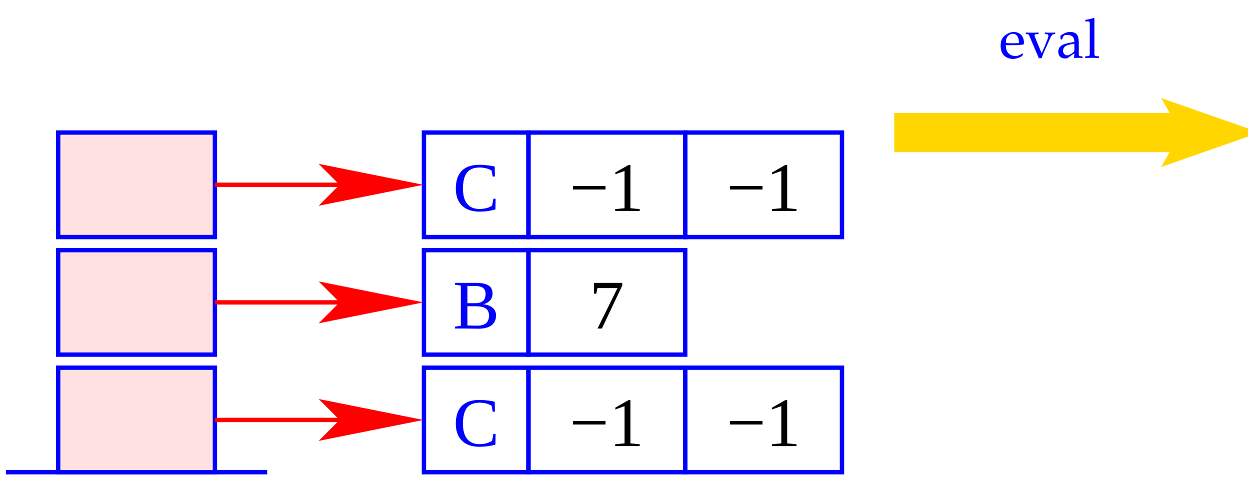
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |



| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | alloc 2 | 3 | rewrite 2 | 3 | mkbasic | 2 | pushloc 1 |
| 2 | pushloc 0 | 2 | loadc 7 | 3 | rewrite 1 | 3 | eval |
| | | | | | | 3 | slide 2 |

Segmentation Fault !!

Apparently, this optimization was not quite correct :-(

## The Problem:

Binding of variable $y$ to variable $x$ before $x$'s dummy node is replaced!!

$$\Longrightarrow$$

## The Solution:

**cyclic definitions:** reject sequences of definitions like

$\textbf{let } a = b; \ldots b = a \textbf{ in } \ldots.$

**acyclic definitions:** order the definitions $y = x$ such that the dummy node for the right side of $x$ is already overwritten.

## Functions:

Functions are values, which are not evaluated further. Instead of generating code that constructs a closure for an F-object, we generate code that constructs the F-object directly.

Therefore:

$$\text{code}_C\ (\mathbf{fn}\ x_0, \ldots, x_{k-1} \Rightarrow e)\ \rho\ \text{sd} \quad = \quad \text{code}_V\ (\mathbf{fn}\ x_0, \ldots, x_{k-1} \Rightarrow e)\ \rho\ \text{sd}$$

# 23 The Translation of a Program Expression

Execution of a program $e$ starts with

$$\mathrm{PC} = 0 \qquad \mathrm{SP} = \mathrm{FP} = \mathrm{GP} = -1$$

The expression $e$ must not contain free variables.

The value of $e$ should be determined and then a halt instruction should be executed.

$$\begin{array}{rcl} \mathrm{code}\ e & = & \mathrm{code}_V\ e\ \emptyset\ 0 \\ & & \mathrm{halt} \end{array}$$

## Remarks:

- The code schemata as defined so far produce Spaghetti code.
- Reason: Code for function bodies and closures placed directly behind the instructions mkfunval resp. mkclos with a jump over this code.
- Alternative: Place this code somewhere else, e.g. following the halt-instruction:

  **Advantage:** Elimination of the direct jumps following mkfunval and mkclos.

  **Disadvantage:** The code schemata are more complex as they would have to accumulate the code pieces in a Code-Dump.

$\Longrightarrow$

## Solution:

Disentangle the Spaghetti code in a subsequent optimization phase :-)

**Example:** $\textbf{let } a = 17;\ f = \textbf{fn } b \Rightarrow a + b \textbf{ in } f\ 42$

Disentanglement of the jumps produces:

```
0      loadc 17          2   mark B          3   B:  slide 2         1   pushloc 1
1      mkbasic           5   loadc 42        1       halt            2   eval
1      pushloc 0         6   mkbasic         0   A:  targ 1          2   getbasic
2      mkvec 1           6   pushloc 4       0       pushglob 0      2   add
2      mkfunval A        7   eval            1       eval            1   mkbasic
                         7   apply           1       getbasic        1   return 1
```