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Program Optimisation

Winter Semester 2004

1. Homework *Deadline: 2 Nov 2004 12:00*

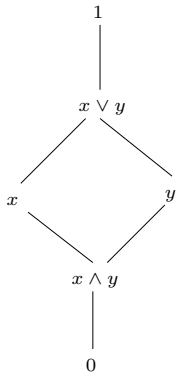
Exercise 1: *6 Points*

Consider the control-flow graph of the function `swap` in the introduction.

a) For each program point u, determine the set $A[u]$ of available expressions at u.
b) Apply the transformations 1 and 2 of the lecture for reusage of expressions.

Exercise 2: *6 Points*

Consider the complete lattice M of monotone Boolean functions in two variables:



a) Give the set of all monotone functions from the lattice M to the complete lattice $\{0, 1\}$ with $0 < 1$.
b) Give the ordering of these functions.

Exercise 3: *6 Points*

Show:

a) If $\mathbb{D}_1$ and $\mathbb{D}_2$ are complete lattices then so is

$$\mathbb{D}_1 \times \mathbb{D}_2 = \{(x, y) \mid x \in \mathbb{D}_1, y \in \mathbb{D}_2\}$$

where $(x_1, y_1) \sqsubseteq (x_2, y_2)$ if and only if $x_1 \sqsubseteq x_2$ and $y_1 \sqsubseteq y_2$.

b) A function $f : \mathbb{D}_1 \times \mathbb{D}_2 \to \mathbb{D}$ is monotone if and only if each of the functions

$$f_x : \mathbb{D}_2 \to \mathbb{D} \quad f_x(y) = f(x, y) \quad (x \in \mathbb{D}_1)$$
$$f_y : \mathbb{D}_1 \to \mathbb{D} \quad f_y(x) = f(x, y) \quad (y \in \mathbb{D}_2)$$

are monotone.

Exercise 4: *6 Points*

Let $\mathbb{D}$ be a complete lattice. For a function $f : \mathbb{D} \to \mathbb{D}$, define the function $f^* : \mathbb{D} \to \mathbb{D}$ as $f^*(x) = \bigsqcup\{f^i(x) \mid i \geq 0\}$. How does f^* look for

a) $f(x) = (x \cap a) \cup b$ ($\mathbb{D} = 2^U$ for a set U)
b) $f(x) = x + 1$ ($\mathbb{D} = \mathbb{N} \cup \{\infty\}$)
c) $f(x) = 2x$ ($\mathbb{D} = \mathbb{N} \cup \{\infty\}$)