Technische Universität München | *Dr. K. N. Verma*
Fakultät für Informatik | *verma@in.tum.de*
Prof. Dr. H. Seidl | *Room: MI 02.07.041*

Program Optimisation

Winter Semester 2004

2. Homework — *Deadline: 9 Nov 2004 12:00*

Exercise 1: *6 Points*

For a complete lattice $\mathbb{D}$, let $h(\mathbb{D}) = n$ be the maximal length of any strictly ascending chain $\bot \sqsubset d_1 \sqsubset \ldots \sqsubset d_n$. Show that for complete lattices $\mathbb{D}_1$ and $\mathbb{D}_2$:

a) $h(\mathbb{D}_1 \times \mathbb{D}_2) = h(\mathbb{D}_1) + h(\mathbb{D}_2)$
b) $h({\mathbb{D}_1}^k) = k \cdot h(\mathbb{D}_1)$
c) $h([\mathbb{D}_1 \to \mathbb{D}_2]) = |\mathbb{D}_1| \cdot h(\mathbb{D}_2)$ where $[\mathbb{D}_1 \to \mathbb{D}_2]$ is the set of monotone functions $f : \mathbb{D}_1 \to \mathbb{D}_2$, and $|\mathbb{D}_1|$ is the cardinality of $\mathbb{D}_1$.

Exercise 2: *6 Points*

Extend the analysis and transformation for available expressions so that the availability of `load` operations are also considered.

Exercise 3: *6 Points*

Consider a constraint system of the form:

$x_i \sqsupseteq f_i(x_{i+1}) \quad (1 \leq i \leq n)$

where each f_i is a monotone function. Show that:

a) The fixpoint-iteration terminates in at most n iterations.
b) One Round-Robin-iteration suffices for a suitable ordering of variables.
c) Can the upper bound n on the maximum number of iterations be reached?