Technische Universität München — *Dr. K. N. Verma*
Fakultät für Informatik — *verma@in.tum.de*
Prof. Dr. H. Seidl — *Room: MI 02.07.041*

Program Optimisation

Winter Semester 2004

9. Homework — *Deadline: 18 Jan 2005 12:00*

Exercise 1: *6 Points*

Consider the following set of operators and constants.

$\mathsf{BinOp} = \{:, +\}$
$\mathsf{UnOp} = \{\mathsf{M}\}$
$\mathsf{Leaf} = \{\mathsf{int}, []\}$

Further assume that we treat non-terminals ("registers") as leaves. Give regular tree-grammars for the following sets of trees.

a) all lists (inner nodes: ":") of int's with an even number of elements (in particular the leftmost node is "[]").
b) all trees in which an M always has, directly below itself, a "+".
c) all trees in which an M never has (directly or indirectly) below itself a ":".

Exercise 2: *6 Points*

For a grammar G let $L(G,R)$ be the set of terminal trees derivable from R.

a) Show that $L(G,R) \neq \emptyset$ if G contains the following rules:

$$R \to a(A,B)$$
$$A \to b(A)$$
$$A \to c(B)$$
$$B \to d$$

b) Give a (if possible, linear) algorithm which, given a grammar G and a non-terminal R, decides whether $L(G,R)$ is non-empty.

Exercise 3: *6 Points*

Let G be a regular grammar of size n and R a non-terminal of G. Show

a) $L(G,R) \neq \emptyset$ iff $t \in L(G,R)$ for some t of depth $\leq n$.
b) $L(G,R)$ is infinite iff $t \in L(G,R)$ for some t of depth d with $n < d \leq 2n$.

(In particular define "size of a grammar" so that these claims hold :-)