Program Optimisation
Solutions of Homework 11

1. (a) A straightforward solution is to proceed from the leaves (level 1) to the root (level d), level by level. Since the block instructions on each level i are independent, they can be used to fill the k slots of VLIW instructions. We need thus at least $\lceil \frac{N}{k} \rceil$ VLIW completely filled instructions. Supplementary, we might need on each level i with $1 \leq i < d$ one partially filled instruction.

   Thus, the total number of VLIW instructions is in the worst case:

$$\lceil \frac{N}{k} \rceil + d - 1 < \frac{N}{k} + 1 + d - 1 = \frac{N}{k} + d$$

   (b) In the previous approach, in the worst case (in which n_i, the number of nodes on each level i, is such that $n_i \bmod k = 1$), we waste on each level except on the root level $k-1$ slots.

   We can improve on this, by filling possible empty slots in the incomplete VLIW instruction for level i with independent block instructions from level $i+1$ for all $1 \leq i < d$ (level d, the root, needs alone one VLIW instruction). In the best case, we can fill all such empty slots. We obtain a number of VLIW instructions equal to:

$$\frac{N-1}{k} + 1$$

   We need to show thus that $\frac{N}{k} + d \leq 2(\frac{N-1}{k} + 1)$.

$$\frac{N}{k} + d \leq 2(\frac{N-1}{k} + 1) \Leftrightarrow$$

$$\frac{N-2}{k} \geq d - 2 \Leftrightarrow$$

$$\frac{N-1-1}{k} \geq d - 2$$

   To be able to fully fill the slots, there must be at least k nodes on each level i with $1 \leq i < d$ (Otherwise, if $n_i < k$, since any instruction of level $i+1$ is dependent on some instruction of level i, we would not be able to place the two instructions in one VLIW.).

   It follows that $N - 1 > k \cdot d$ and thus:

$$\frac{N-1-1}{k} \geq \frac{k \cdot d - 1}{k} = d - \frac{1}{k}$$

Further:

$$d - \frac{1}{k} \geq d - 2 \Leftrightarrow k \geq 1$$

It follows q.e.d.