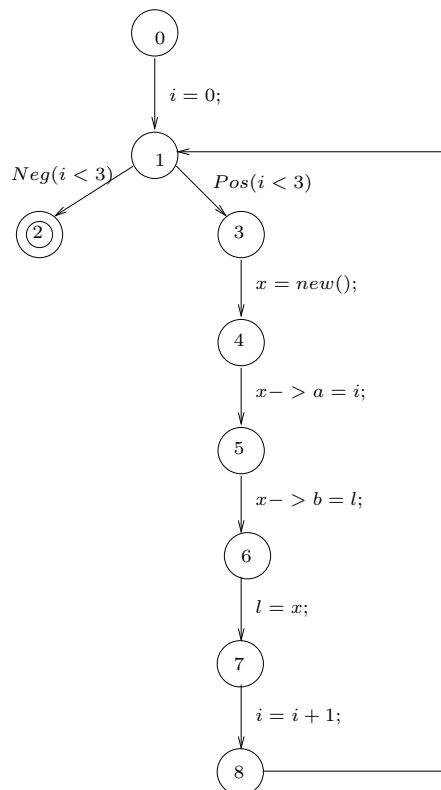# Program Optimisation
Solutions of Homework 4

1. Here is the CFG for the given program.

For the first analysis, we have the following constraints from the program instructions.

$$
\begin{aligned}
\mathcal{P}[x] &\supseteq \{(3,4)\} \\
\mathcal{P}[f.a] &\supseteq \mathcal{P}[i] && \text{for all } f \in \mathcal{P}[x] \\
\mathcal{P}[f.a] &\supseteq \mathcal{P}[l] && \text{for all } f \in \mathcal{P}[x] \\
\mathcal{P}[l] &\supseteq \mathcal{P}[x]
\end{aligned}
$$

It gives us the following solution:

x	$\{(3,4)\}$
l	$\{(3,4)\}$
i	$\{\}$
$(3,4).a$	$\{\}$
$(3,4).b$	$\{(3,4)\}$

For the second analysis we start with the following partition:

$$\{\{x\}, \{l\}, \{i\}, \{x->a\}, \{x->b\}, \{l->a\}, \{l->b\}, \{i->a\}, \{i->b\}\}$$

The instruction $x->a = i;$ leads us to the following partition:

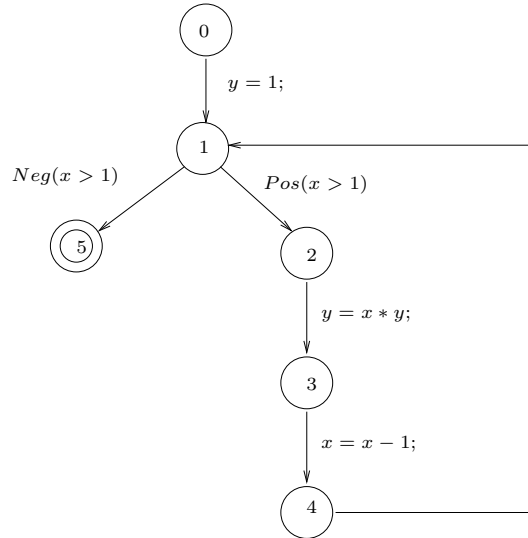$$\{\{x\}, \{l\}, \{i, x->a\}, \{x->b\}, \{l->a\}, \{l->b\}, \{i->a\}, \{i->b\}\}$$

The instruction $x->b = l;$ leads us to the following partition:

$$\{x\}, \{l, x->b\}, \{i, x->a\}, \{l->a\}, \{l->b\}, \{i->a\}, \{i->b\}\}$$

The instruction $l = x$ leads us to the following partition:

$$\{x, l, x->b, l->b\}, \{i, x->a, l->a\}, \{i->a\}, \{i->b\}\}$$

2. Here is the CFG for the factorial program.

The set of expressions is $Expr = \{1, x > 1, x * y, x - 1\}$. Here are the constraints for the computation of available expressions:

$$\mathcal{A}[0] \subseteq \{\}$$
$$\mathcal{A}[1] \subseteq ((\mathcal{A}[0] \cup \{1\}) \setminus Expr_y) \cap \mathcal{A}[4]$$
$$\mathcal{A}[2] \subseteq \mathcal{A}[1] \cup \{x > 1\}$$
$$\mathcal{A}[3] \subseteq (\mathcal{A}[2] \cup \{x * y\}) \setminus Expr_y$$
$$\mathcal{A}[4] \subseteq (\mathcal{A}[3] \cup \{x - 1\}) \setminus Expr_x$$
$$\mathcal{A}[5] \subseteq \mathcal{A}[1] \cup \{x > 1\}$$

From these constraints we obtain the following dependence relation:

	I
0	$\{1\}$
1	$\{2,5\}$
2	$\{3\}$
3	$\{4\}$
4	$\{1\}$
5	$\{\}$

The workset iteration works as follows:

$D[0]$	$D[1]$	$D[2]$	$D[3]$	$D[4]$	$D[5]$	W
$Expr$	$Expr$	$Expr$	$Expr$	$Expr$	$Expr$	$\{\boxed{0},1,2,3,4,5\}$
$\{\}$	$Expr$	$Expr$	$Expr$	$Expr$	$Expr$	$\{\boxed{1},2,3,4,5\}$
$\{\}$	$\{1\}$	$Expr$	$Expr$	$Expr$	$Expr$	$\{\boxed{2},3,4,5\}$
$\{\}$	$\{1\}$	$\{1, x>1\}$	$Expr$	$Expr$	$Expr$	$\{\boxed{3},4,5\}$
$\{\}$	$\{1\}$	$\{1, x>1\}$	$\{1, x>1\}$	$Expr$	$Expr$	$\{\boxed{4},5\}$
$\{\}$	$\{1\}$	$\{1, x>1\}$	$\{1, x>1\}$	$\{1\}$	$Expr$	$\{\boxed{1},5\}$
$\{\}$	$\{1\}$	$\{1, x>1\}$	$\{1, x>1\}$	$\{1\}$	$Expr$	$\{\boxed{5}\}$
$\{\}$	$\{1\}$	$\{1, x>1\}$	$\{1, x>1\}$	$\{1\}$	$\{1, x>1\}$	$\{\}$

The number of calculations on the right hand side is 7.