## Program Optimisation Solutions of Homework 5

1. The CFG of the given program is in Abbildung 1.


Abbildung 1: CFG of the given program

We perform loop rotation and introduce registers $T_{e}$ for expressions $e$. The result is shown in Abbildung 2.

We are interested in the set of expressions Expr $=\{a+2\}$. The set of available and very busy expressions at each program point is as below:

| $u$ | $\mathcal{A}[u]$ | $\mathcal{B}[u]$ |
| :---: | :---: | :---: |
| 9 | $\}$ | $\}$ |
| 8 | $\{a+2\}$ | $\}$ |
| 7 | $\{a+2\}$ | $\}$ |
| 6 | $\{a+2\}$ | $\}$ |
| 5 | $\{a+2\}$ | $\}$ |
| 4 | $\{a+2\}$ | $\}$ |
| 3 | $\{a+2\}$ | $\}$ |
| 2 | $\}$ | $\{a+2\}$ |
| 1 | $\}$ | $\}$ |
| 0 | $\}$ | $\}$ |



Abbildung 2: After loop rotations

We apply Transformation 6.1 and Transformation 6.2 of the lecture to obtain the CFG in Abbildung 3.

In case the statement if ( $\mathrm{j}>\mathrm{i}$ ) ... is at the beginning of the body of the loop then instead of Abbildung 2 we have Abbildung 4.
Then we may apply Transformation 7 to obtain the CFG in Abbildung 5.
Then as before we can do the loop invariant computation.
2. a) Let the program be loop dominated. Let $I$ be the set of the unique entry points of the loops in the program. By definition of $I, I$ contains a point from every loop (namely the entry point of the loop). Hence by definition of loop separators, $I$ is a loop separator for the program.
b) Transformation of the loop of the example program for interval-analysis into a do-while-loop leads to the program in Abbildung 6.
c) Interval analysis without narrowing on this program works as follows. We use the loop separator $I=\{2\}$. So widening is applied only at point 2 .


Abbildung 3: After transformations 6.1 and 6.2


Abbildung 4: CFG with 'if-break' statement at the beginning of loop


Abbildung 5: After transformation 7

|  | 1 |  | 2 |  | 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $l$ | $u$ | $l$ | $u$ | $l$ | $u$ |
| 0 | $-\infty$ | $+\infty$ | $-\infty$ | $+\infty$ |  |  |
| 1 | 0 | 0 | 0 | 0 |  |  |
| 2 | 0 | 0 | 0 | $+\infty$ |  |  |
| 3 | 0 | 0 | 0 | 41 |  |  |
| 4 | 0 | 0 | 0 | 41 | dito |  |
| 5 | 0 | 0 | 0 | 41 |  |  |
| 6 | 1 | 1 | 1 | 42 |  |  |
| 7 | 1 | 1 | 1 | 41 |  |  |
| 8 |  | $\perp$ | 42 | $+\infty$ |  |  |
| 9 |  | $\perp$ | 42 | 42 |  |  |



Abbildung 6: After loop rotation

