Program Optimisation
Solutions of Homework 6

1. First observe that the polynomial $x^n - (x-1)^n$ is of degree $n-1$. This is because

$$\begin{aligned} x^n - (x-1)^n &= x^n - \binom{n}{0}x^n + \binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2} + \ldots + \binom{n}{n-1}x + \binom{n}{n} \\ &= \binom{n}{1}x^{n-1} + \ldots \end{aligned}$$

   This also means that if $p(x)$ is a polynomial of degree n, then $p(x) - p(x-1)$ is of degree $n-1$. To show this consider

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

   where $a_n \neq 0$. We have

$$p(x)-p(x-1) = a_n(x^n-(x-1)^n)+a_{n-1}(x^{n-1}-(x-1)^{n-1})+\ldots+a_1(x-(x-1))+a_0(1-1)$$

   and we know that $x^n - (x-1)^{n-1}$ is of degree $n-1$, $x^{n-1} - (x-1)^{n-1}$ is of degree $n-2$, ... Hence $p(x) - p(x-1)$ is of degree $n-1$.

   Now consider given a polynomial $p(x)$ is degree k.
   Hence $\Delta_0(x) = p(x)$ is of degree k.
   Hence $\Delta_1(x) = \Delta_0(x) - \Delta_0(x-1)$ is of degree $k-1$.
   Hence $\Delta_2(x) = \Delta_1(x) - \Delta_1(x-1)$ is of degree $k-2$.
   Hence $\Delta_3(x) = \Delta_2(x) - \Delta_2(x-1)$ is of degree $k-3$.
   ...
   Hence $\Delta_k(x) = \Delta_{k-1}(x) - \Delta_{k-1}(x-1)$ is of degree $k-k=0$.
   Hence $\Delta_k(x)$ is a constant.

2. We are given the following program:

```
B = b;
for (i=0; i<n; i++) {
     A = a + i*h;
     for (j=0; j<h; j++) {
          A1 = A + j;
          B1 = B + j;
          T = M[A1];
          M[B1] = T;
     }
     B = b + i*h;
}
```

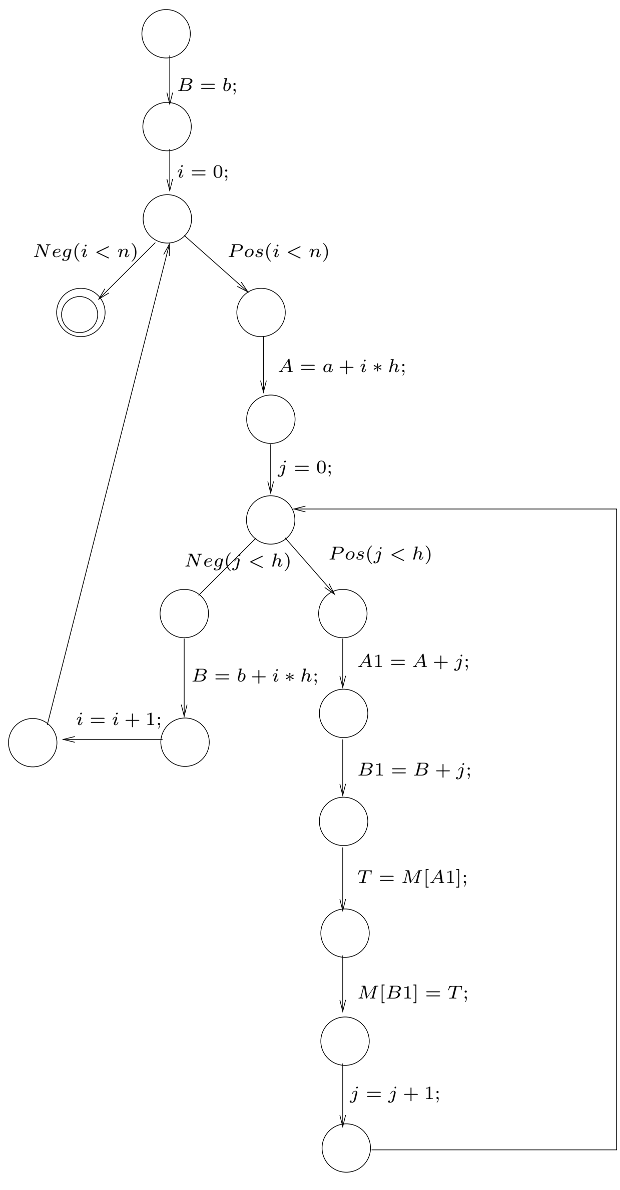

Abbildung 1: CFG of given program

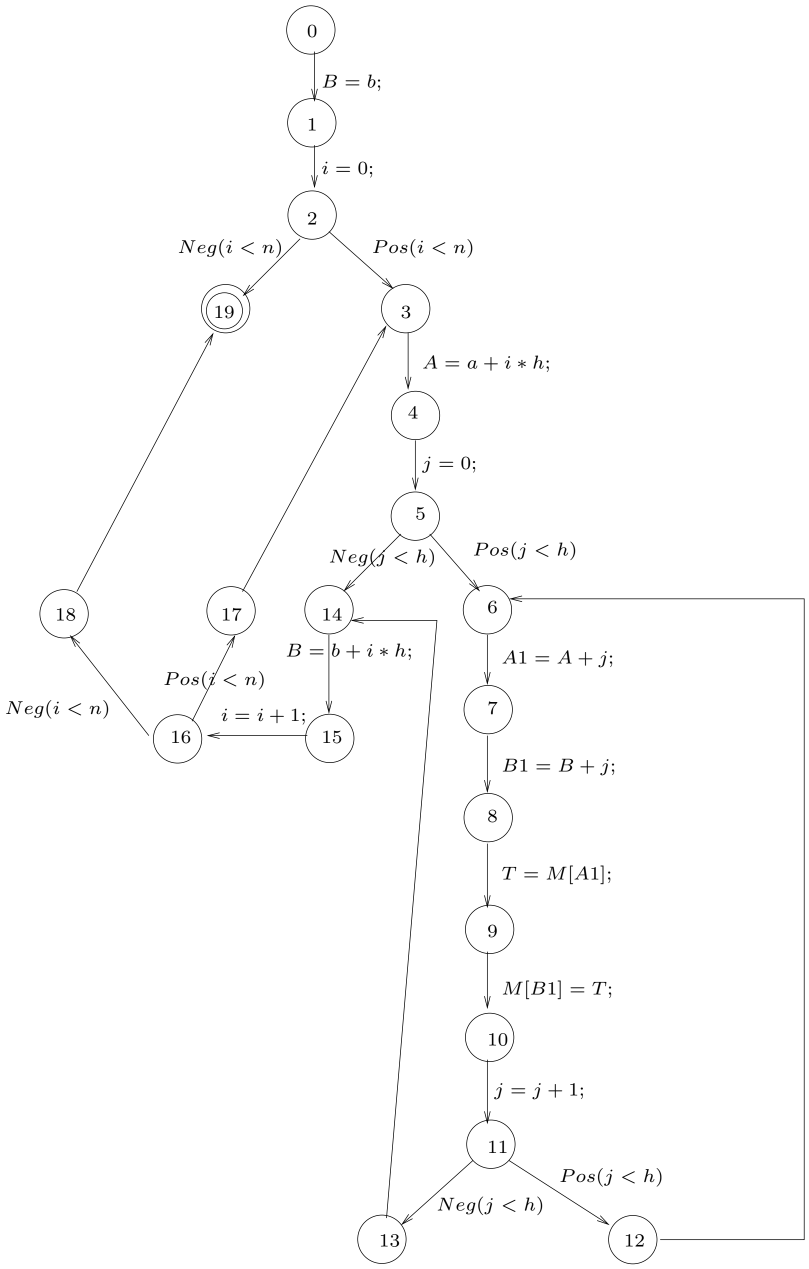

Abbildung 2: After loop rotations

The corresponding CFG is in Abbildung 1.

Loop rotations produce the CFG in Abbildung 2.

The inner loop involves iteration variable j. $j, A1$ and $B1$ are modified exactly once. A and B are not modified in the loop. Hence we apply the transformation to obtain the CFG in Abbildung 3.

The outer loop involves iteration variable i. i, A and B are modified exactly once. h, a and b are not modified in the loop. We apply the transformation on this loop to obtain the CFG in Abbildung 4. Observe that we have applied ROS for the modifications to A but not the modifications to B. This is because the modification to B is at the end at the loop and the initial value $B = b$ happens to be used in the iterations of the inner loop in the first iteration of the outer loop. Hence our ROS transformation would give the wrong result because we would overwrite the initial value for B. (There are alternative solutions if we insist on optimizing the modifications to B).

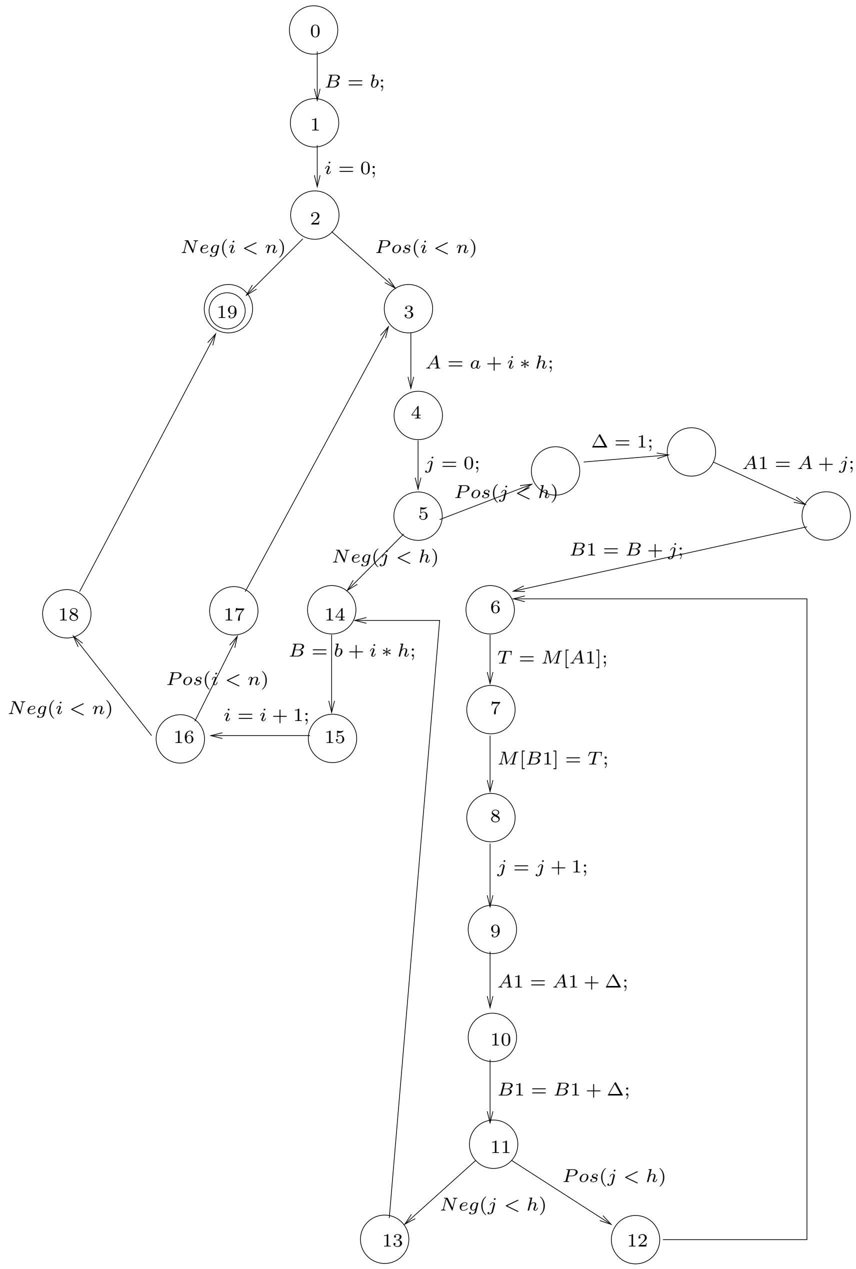

Abbildung 3: After transformation on the inner loop

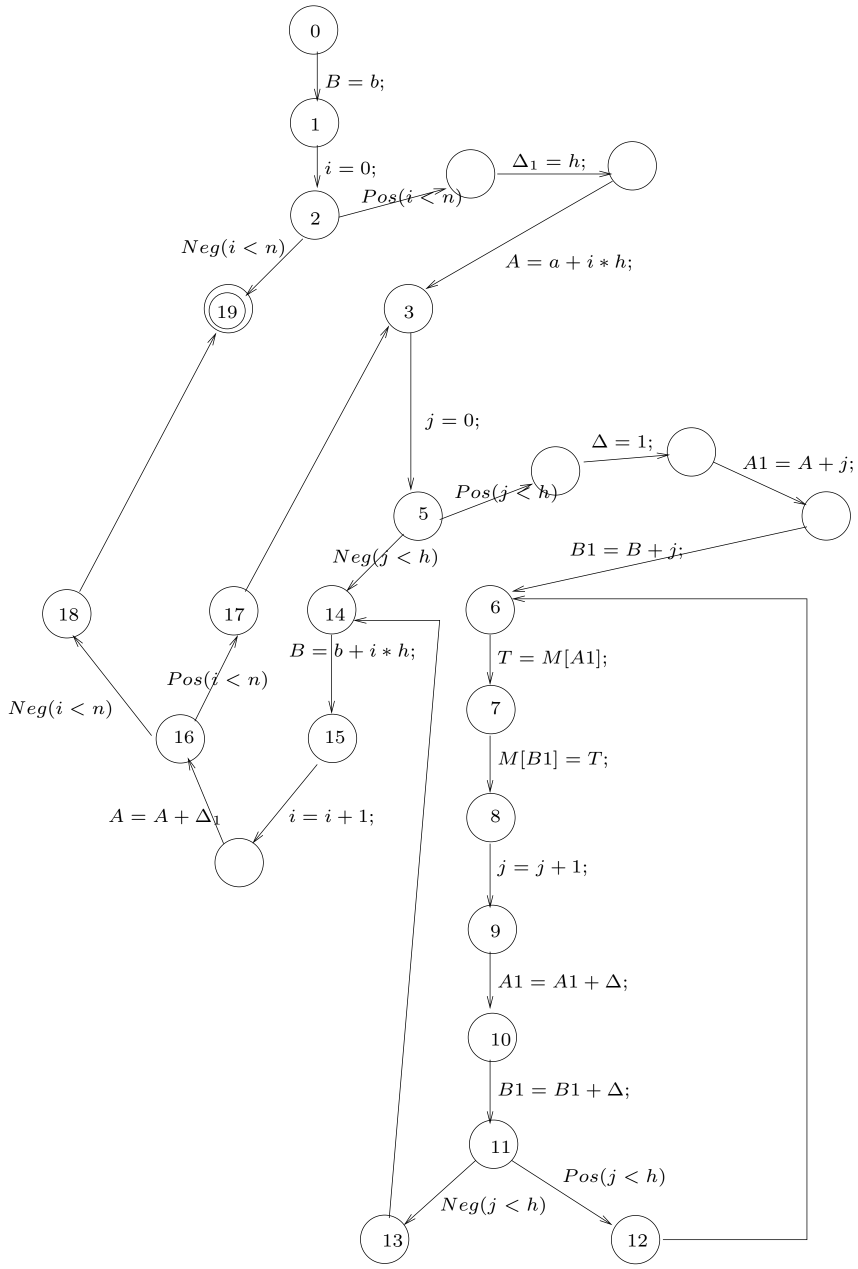

Abbildung 4: After transformation on the inner loop