## Program Optimisation Solutions of Homework 7

1. The function f 1 was originally defined as:
```
f1 (n,x,y) {
    if (n <= 1) return y;
    n = n-1;
    z = x+y;
    z = f1 (n,y,z);
    return z;
}
```

We eliminate tail recursion from it to produce the following function:

```
        f1 (n,x,y) {
_f1: if (n <= 1) return y;
            n = n-1;
        z = x+y;
        n = n;
        x = y;
        y = z;
        goto _f1;
        }
```

The function $f$ was originally defined as:

```
f (n) {
    z = f1 (n,1,1);
    return z;
}
```

As the call to $f 1$ is at the end of the function body, we can eliminate it, similar to the previous case, by copying the body of the function $f 1$. The new definition of $f$ is as follows:

$$
\begin{aligned}
& f(n)\{ \\
& n=n ;
\end{aligned}
$$

```
    x = 1;
    y = 1;
_f1: if (n <= 1) return y;
    n = n-1;
    z = x+y;
    n = n;
    x = y;
    y = z
    goto _f1;
}
```

Finally here is the original definition of the function main.

```
main () {
    n = read ();
    y = f (n);
    write (y);
}
```

We eliminate the call to f to produce the following definition.

```
    main () {
    n = read ();
    n = n;
    n = n;
    x = 1;
    y = 1;
_f1: if (n <= 1) {y = y; goto _f;}
    n = n-1;
    z = x+y;
    n = n;
    x = y;
    y = z;
    goto _f1;
_f: write (y)
}
```

