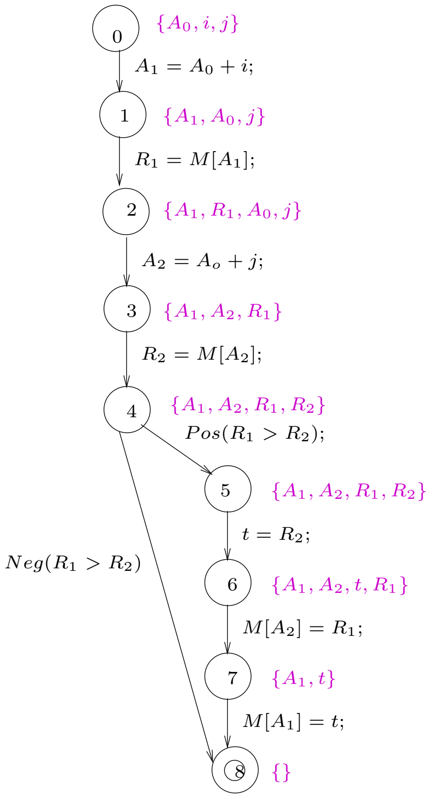
Program Optimisation
Solutions of Homework 8

1. Here is the CFG for the optimised version of the version `swap` from the lecture, together withe the live variables at each program point. The program uses 8 variables $A_0, i, j, A_1, A_2, R_1, R_2$ and t.



The corresponding interval graph is shown in Abbildung 1. By looking at it, we can see that four registers are sufficient. We call these registers *red*, *blue*, *green* and *yellow*. The allocation of registers is also shown in the same figure using colors. We can check that at any program point, all intervals have distinct colors (registers).

2. a) Suppose G has no loop. We show that any connected component G' of G can be colored using at most 2 colors. Since G' is connected and has no loops, it is a tree. Let $V_0 = \{v\}$ where v is the root node. For $i > 0$ let V_i be the set of successors of node in V_i. We color each node in V_i as red if i is even, and as blue if i is odd.

   b) Suppose each node in G has degree at most 2. We show that each connected component G' of G can be colored using at most 3 colors. If G' has no loops

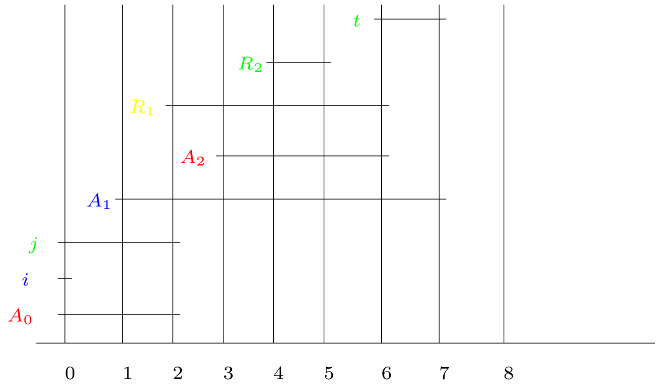


Abbildung 1: Interval graph and allocation of registers

then by the previous part, we know that G' can be colored using at most 2 colors.
If G' has a loop then consider a minimal loop in G'. I.e. we consider pairwise distinct nodes $v_0, v_1, \ldots, v_n$ for some $n \geq 1$ such that v_i is neighbor of v_{i+1} for $0 \leq i \leq n-1$, and v_n is a neighbor of v_0.
We claim that G' has only these $n+1$ nodes and $n+1$ edges. Otherwise some v_i would be a neighbor of some node v, where $v \neq v_j$ for any j. But then v_i would have degree at least 3 leading to contradiction.
Now for $0 \leq i \leq n-1$, if i is even then we color v_i as red, otherwise we color v_i as blue. The last node v_n is colored green.

c) If G has a k-clique, then we have k distinct nodes $v_1, \ldots, v_k$ such that for any $1 \leq i, j \leq k$ with $i \neq j$, v_i and v_j are neighbors. Hence v_i and v_j should be colored with different colors. In other words, each of the k nodes should have a different color. Hence we required at least k colors for coloring this graph.