TECHNISCHE UNIVERSITÄT MÜNCHEN
FAKULTÄT FÜR INFORMATIK

Lehrstuhl für Sprachen und Beschreibungsstrukturen WS 2008/09
Einführung in die Informatik 2 **Übungsblatt 2**
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Abgabe: 28.10.08 (vor der Vorlesung)

Aufgabe 2.1 (H) Logik

Zeigen oder widerlegen Sie die Allgemeingültigkeit folgender Aussagen. Ist eine Aussage A nicht allgemeingültig, so ist ein Zustand σ anzugeben, so dass $\sigma \not\models A$ gilt.

a) $\mathtt{x} = \mathtt{y} + 1 \wedge \mathtt{y} = (\mathtt{z} + 1)(\mathtt{z} - 1) \Rightarrow \mathtt{x} \geq 0$

b) $\mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5 \Rightarrow (\neg(\mathtt{i} \neq \mathtt{n}) \wedge \mathtt{x} = 10 \cdot \mathtt{n}) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5)$

c) $\mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5 \Leftarrow (\neg(\mathtt{i} \neq \mathtt{n}) \wedge \mathtt{x} = 10 \cdot \mathtt{n}) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5)$

d)

$$\begin{aligned} & \mathtt{m}^2 = -4 \cdot \mathtt{m} - 4 \wedge ((\neg(\mathtt{i} < \mathtt{n}) \wedge \mathtt{i} \leq \mathtt{n} \wedge \mathtt{n} = \mathtt{m} \cdot \mathtt{k}) \vee (\mathtt{i} < \mathtt{n} \wedge \mathtt{i} \leq \mathtt{n} \wedge \mathtt{i} = \mathtt{m} \cdot \mathtt{k})) \\ \equiv\ & \mathtt{m} = -2 \wedge \mathtt{i} \leq \mathtt{n} \wedge \mathtt{i} = -2 \cdot \mathtt{k} \end{aligned}$$

Lösungsvorschlag 2.1

a)

$$\begin{aligned} \mathtt{x} = \mathtt{y} + 1 \wedge \mathtt{y} = (\mathtt{z} + 1)(\mathtt{z} - 1) &\equiv \mathtt{x} = \mathtt{y} + 1 \wedge \mathtt{y} = \mathtt{z}^2 - 1 \\ &\equiv \mathtt{x} = \mathtt{z}^2 - 1 + 1 \wedge \mathtt{y} = \mathtt{z}^2 - 1 \\ &\equiv \mathtt{x} = \mathtt{z}^2 \wedge \mathtt{y} = \mathtt{z}^2 - 1 \\ &\Rightarrow \mathtt{x} = \mathtt{z}^2 \\ &\Rightarrow \mathtt{x} \geq 0 \end{aligned}$$

b)

$$\begin{aligned} & (\neg(i \neq n) \wedge x = 10 \cdot n) \vee (i \neq n \wedge x = 10 \cdot i \wedge k = 5) \\ \equiv\ & (\mathtt{i} = \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{n}) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \\ \equiv\ & (\mathtt{i} = \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i}) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \\ \Leftarrow\ & (\mathtt{i} = \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \\ \equiv\ & (\mathtt{i} = \mathtt{n} \vee \mathtt{i} \neq \mathtt{n}) \wedge (\mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \\ \equiv\ & \mathbf{true} \wedge (\mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5) \\ \equiv\ & \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5 \end{aligned}$$

c) Die Aussage ist nicht allgemeingültig. Gegenbeispiel: Es gilt:

$$\begin{aligned} & \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5 \Leftarrow (\neg(\mathtt{i} \neq \mathtt{n}) \wedge \mathtt{x} = 10 \cdot \mathtt{n}) \vee (\mathtt{i} \neq \mathtt{n} \wedge \mathtt{x} = 10 \cdot \mathtt{i} \wedge \mathtt{k} = 5)[0/\mathtt{i}, 0/\mathtt{n}, 0/\mathtt{x}, 0/\mathtt{k}] \\ \equiv\ & 0 = 10 \cdot 0 \wedge 0 = 5 \Leftarrow (\neg(0 \neq 0) \wedge 0 = 10 \cdot 0) \vee (0 \neq 0 \wedge 0 = 10 \cdot 0 \wedge 0 = 5) \\ \equiv\ & \mathbf{false} \Leftarrow (\mathbf{true} \wedge \mathbf{true}) \vee (\mathbf{false} \wedge \mathbf{true} \wedge \mathbf{false}) \\ \equiv\ & \mathbf{false} \Leftarrow (\mathbf{true} \wedge \mathbf{true}) \vee \mathbf{false} \\ \equiv\ & \mathbf{false} \Leftarrow \mathbf{true} \\ \equiv\ & \mathbf{false} \end{aligned}$$

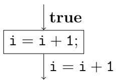
d)

$$
\begin{aligned}
& \mathtt{m}^2 = -4\cdot\mathtt{m} - 4 \land ((\neg(\mathtt{i} < \mathtt{n}) \land \mathtt{i} \leq \mathtt{n} \land \mathtt{n} = \mathtt{m}\cdot\mathtt{k}) \lor (\mathtt{i} < \mathtt{n} \land \mathtt{i} \leq \mathtt{n} \land \mathtt{i} = \mathtt{m}\cdot\mathtt{k}))\\
\equiv\; & \mathtt{m}^2 + 4\cdot\mathtt{m} + 4 = 0 \land ((\mathtt{i} \geq \mathtt{n} \land \mathtt{i} \leq \mathtt{n} \land \mathtt{n} = \mathtt{m}\cdot\mathtt{k}) \lor (\mathtt{i} < \mathtt{n} \land \mathtt{i} = \mathtt{m}\cdot\mathtt{k}))\\
\equiv\; & (\mathtt{m}+2)^2 = 0 \land ((\mathtt{i} = \mathtt{n} \land \mathtt{n} = \mathtt{m}\cdot\mathtt{k}) \lor (\mathtt{i} < \mathtt{n} \land \mathtt{i} = \mathtt{m}\cdot\mathtt{k}))\\
\equiv\; & \mathtt{m} = -2 \land ((\mathtt{i} = \mathtt{n} \land \mathtt{i} = -2\cdot\mathtt{k}) \lor (\mathtt{i} < \mathtt{n} \land \mathtt{i} = -2\cdot\mathtt{k}))\\
\equiv\; & \mathtt{m} = -2 \land (\mathtt{i} = \mathtt{n} \lor \mathtt{i} < \mathtt{n}) \land \mathtt{i} = -2\cdot\mathtt{k}\\
\equiv\; & \mathtt{m} = -2 \land \mathtt{i} \leq \mathtt{n} \land \mathtt{i} = -2\cdot\mathtt{k}
\end{aligned}
$$

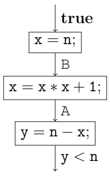
Aufgabe 2.2 (H) Verifikation

Überprüfen Sie, ob die Annotationen in folgenden Kontrollfluß-Diagrammen lokal konsistent sind. Unter Umständen müssen fehlende Zusicherungen (mit `A`, `B` und `C` bezeichnet) ergänzt werden. Falls lokale Konsistenz gegeben ist, dann ist ein Beweis dafür anzugeben. Andernfalls ist anzugeben an welcher Stelle die lokale Konsistenz verletzt ist. Zusätzlich ist ein Zustand σ zu nennen, der die Verletzung aufzeigt.

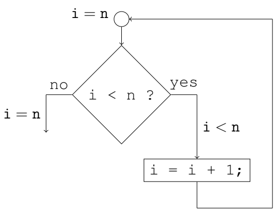
a)



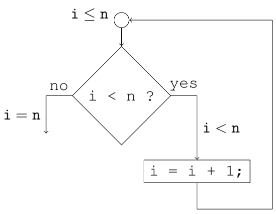
b)



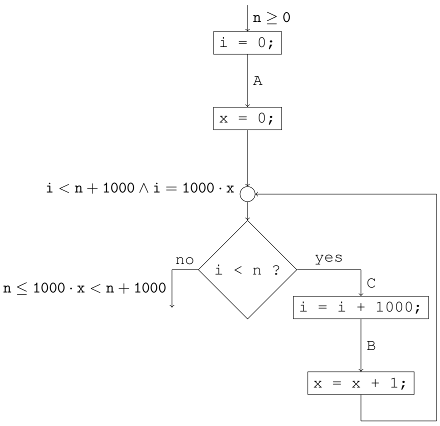
c)



d)



e)



Lösungsvorschlag 2.2

a) Die Annotationen sind nicht lokal konsistent, denn

$$
\begin{aligned}
\mathbf{WP}[\![\mathtt{i} = \mathtt{i} + 1]\!](\mathtt{i} = \mathtt{i} + 1) &\equiv (\mathtt{i} = \mathtt{i} + 1)[\mathtt{i} + 1/\mathtt{i}] \\
&\equiv \mathtt{i} + 1 = \mathtt{i} + 2 \\
&\equiv \mathbf{false}
\end{aligned}
$$

und $\mathtt{true} \not\Rightarrow \mathbf{false}$.

b) Die Annotationen sind lokal konsistent, da folgende Aussagen gelten:

$$\mathbf{WP}[\![\mathtt{y = n - x}]\!](\mathtt{y < n}) \equiv (\mathtt{y < n})[\mathtt{n - x/y}] \equiv \mathtt{n - x < n} \Leftarrow \mathtt{x > 0} \equiv: \mathtt{A}$$

$$\mathbf{WP}[\![\mathtt{x = x * x + 1}]\!](\mathtt{A}) \equiv \mathtt{A}[\mathtt{x * x + 1/x}] \equiv \mathtt{x \cdot x + 1 > 0} \equiv \mathbf{true} \equiv: \mathtt{B}$$

$$\mathbf{WP}[\![\mathtt{x = n}]\!](\mathtt{B}) \equiv \mathbf{true}[\mathtt{n/x}] \equiv \mathbf{true}$$

c) Die Annotationen sind nicht lokal konsistent, denn

$$\mathbf{WP}[\![\mathtt{i = i + 1}]\!](\mathtt{i = n}) \equiv (\mathtt{i = n})[\mathtt{i + 1/i}] \equiv \mathtt{i + 1 = n} \not\Leftarrow \mathtt{i < n}$$

gilt, da zum Beispiel

$$\begin{aligned}(\mathtt{i + 1 = n} \not\Leftarrow \mathtt{i < n})[\mathtt{0/i, 2/n}] &\equiv \mathbf{false} \not\Leftarrow \mathbf{true} \\ &\equiv \neg(\mathbf{false} \Leftarrow \mathbf{true}) \\ &\equiv \neg\mathbf{false} \\ &\equiv \mathbf{true}\end{aligned}$$

gilt.

d) Die Annotationen sind lokal konsistent, da folgende Aussagen gelten:

$$\begin{aligned}\mathbf{WP}[\![\mathtt{i = i + 1;}]\!](\mathtt{i \leq n}) &\equiv \mathtt{i + 1 \leq n} \\ &\equiv \mathtt{i < n}\end{aligned}$$

$$\begin{aligned}\mathbf{WP}[\![\mathtt{i < n}]\!](\mathtt{i = n, i < n}) &\equiv (\neg(\mathtt{i < n}) \wedge \mathtt{i = n}) \vee (\mathtt{i < n} \wedge \mathtt{i < n}) \\ &\equiv (\mathtt{i \geq n} \wedge \mathtt{i = n}) \vee \mathtt{i < n} \\ &\equiv \mathtt{i = n} \vee \mathtt{i < n} \\ &\equiv \mathtt{i \leq n}\end{aligned}$$

e) Die Annotationen sind lokal konsistent, da folgende Aussagen gelten:

$$\begin{aligned}\mathbf{WP}[\![\mathtt{x = 0;}]\!](\mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x}) &\equiv \mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot 0} \\ &\equiv \mathtt{i < n + 1000} \wedge \mathtt{i = 0} \\ &\equiv: \mathtt{A}\end{aligned}$$

$$\begin{aligned}\mathbf{WP}[\![\mathtt{i = 0;}]\!](\mathtt{A}) &\equiv \mathtt{0 < n + 1000} \wedge \mathbf{true} \\ &\Leftarrow \mathtt{0 \leq n}\end{aligned}$$

$$\begin{aligned}\mathbf{WP}[\![\mathtt{x = x + 1;}]\!](\mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x}) &\equiv \mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot (x + 1)} \\ &\equiv \mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x + 1000} \\ &\equiv: \mathtt{B}\end{aligned}$$

$$\begin{aligned}\mathbf{WP}[\![\mathtt{i = i + 1000;}]\!](\mathtt{B}) &\equiv \mathtt{i + 1000 < n + 1000} \wedge \mathtt{i + 1000 = 1000 \cdot x + 1000} \\ &\equiv \mathtt{i < n} \wedge \mathtt{i = 1000 \cdot x} \\ &\equiv: \mathtt{C}\end{aligned}$$

Sei $\mathtt{Z} :\equiv \mathtt{n \leq 1000 \cdot x < n + 1000}$. Es ist zu zeigen, dass

$$\mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x} \Rightarrow \mathbf{WP}[\![\mathtt{i < n}]\!](\mathtt{Z,\ C})$$

gilt. Da $\mathbf{WP}[\![\mathtt{i < n}]\!](\mathtt{Z,\ C}) \equiv (\mathtt{i \geq n} \wedge \mathtt{Z}) \vee (\mathtt{i < n} \wedge \mathtt{C})$ gilt, ist zu zeigen, dass

$$\mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x} \Rightarrow (\mathtt{i \geq n} \wedge \mathtt{Z}) \vee (\mathtt{i < n} \wedge \mathtt{C})$$

gilt. Dazu nehmen wir an, dass $\mathtt{i < n + 1000} \wedge \mathtt{i = 1000 \cdot x}$ gilt. Ziel ist es jetzt zu zeigen, dass $(\mathtt{i \geq n} \wedge \mathtt{Z}) \vee (\mathtt{i < n} \wedge \mathtt{C})$ gilt. Wir machen eine Fallunterscheidung.

Fall 1: $\mathtt{i} < \mathtt{n}$. Unter diesen Voraussetzungen gilt also $\mathtt{i} < \mathtt{n} \wedge \mathtt{i} = 1000 \cdot \mathtt{x}$. D.h. es gilt $\mathtt{i} < \mathtt{n} \wedge \mathtt{C}$. Damit gilt $(\mathtt{i} \geq \mathtt{n} \wedge \mathtt{Z}) \vee (\mathtt{i} < \mathtt{n} \wedge \mathtt{C})$.

Fall 2: $\mathtt{i} \geq \mathtt{n}$. Unter diesen Voraussetzungen gilt also $\mathtt{i} \geq \mathtt{n} \wedge \mathtt{i} < \mathtt{n} + 1000 \wedge \mathtt{i} = 1000 \cdot \mathtt{x}$. Damit gilt insbesondere $\mathtt{i} \geq \mathtt{n} \wedge \mathtt{n} \leq 1000 \cdot \mathtt{x} < \mathtt{n} + 1000$. Damit gilt $\mathtt{i} \geq \mathtt{n} \wedge \mathtt{Z}$. Damit gilt $(\mathtt{i} \geq \mathtt{n} \wedge \mathtt{Z}) \vee (\mathtt{i} < \mathtt{n} \wedge \mathtt{C})$.

Aufgabe 2.3 (P) Verifikation

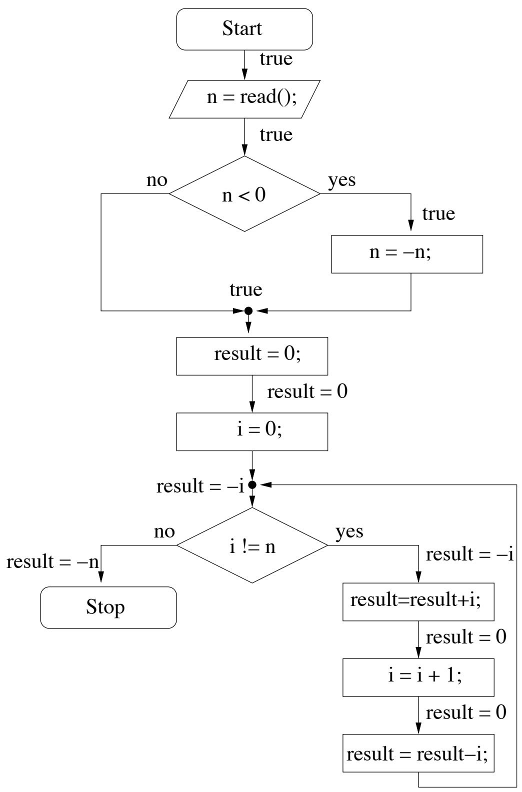
Gegeben sei folgendes MiniJava-Programm:

```
int n, i, result;

n = read();
if (n < 0)
  n = -n;

result = 0;
i = 0;
while (i != n) {
  result = result + i;
  i = i + 1;
  result = result - i;
}
```

a) Erstellen Sie das Kontrollfluss-Diagramm!

b) Zeigen Sie, dass am Stop-Knoten die Zusicherung $\texttt{result} = -\texttt{n}$ stets erfüllt ist.

Lösungsvorschlag 2.3

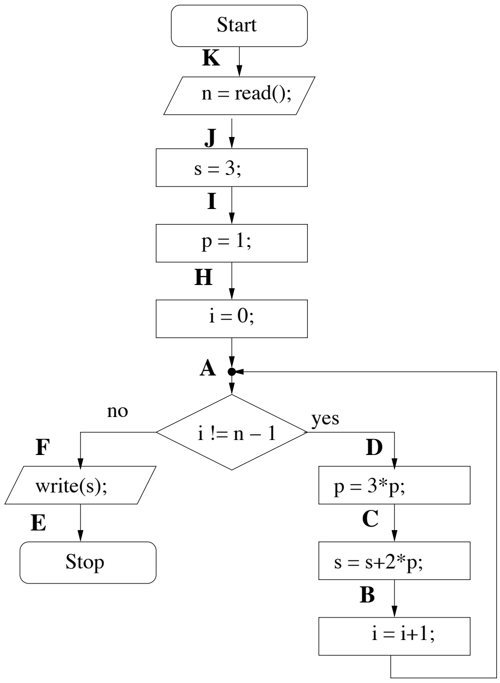


Dabei ist die einzige Stelle, an der die lokale Konsistenz nicht-trivial ist, der Bedingungs-Knoten:

$$\begin{aligned}
&\mathbf{WP}[\![\texttt{i!} = \texttt{n}]\!](\texttt{result} = -\texttt{n}, \texttt{result} = -\texttt{i}) \\
\equiv\ & (\texttt{i} = \texttt{n} \wedge \texttt{result} = -\texttt{n}) \vee (\texttt{i} \neq \texttt{n} \wedge \texttt{result} = -\texttt{i}) \\
\equiv\ & (\texttt{i} = \texttt{n} \wedge \texttt{result} = -\texttt{i}) \vee (\texttt{i} \neq \texttt{n} \wedge \texttt{result} = -\texttt{i}) \\
\equiv\ & (\texttt{i} = \texttt{n} \vee \texttt{i} \neq \texttt{n}) \wedge \texttt{result} = -\texttt{i} \\
\equiv\ & \texttt{result} = -\texttt{i}
\end{aligned}$$

Aufgabe 2.4 (P) Eine alte, vereinfachte Klausuraufgabe

Gegeben sei folgendes Kontrollfluß-Diagramm:



a) Bestimmen Sie alle Zustände, die angenommen werden, falls die Zahl 3 eingegeben wird.

b) Zeigen Sie, dass am Ende des Programms die Zusicherung $\mathtt{s} = 3^{\mathtt{n}}$ stets erfüllt ist.

Hinweis: Als Hilfestellung sei Ihnen folgende Rechenregel gegeben:

$$3^{n+1} = 1 + 2 \cdot \sum_{i=0}^{n} 3^i, \qquad \text{für } n \in \mathbb{N}_{\geq 0}.$$

Lösungsvorschlag 2.4

a) Wir nahmen an, dass im Punkt K mit dem Zustand $\{\mathtt{n} \mapsto x_1, \mathtt{s} \mapsto x_2, \mathtt{p} \mapsto x_3, \mathtt{i} \mapsto x_4\}$ gestartet wird (wobei $x_i \in \mathbb{Z}$ beliebig für $i = 1, \ldots, 4$). Unter der Annahme, dass 3 eingelesen wird, wird folgender Pfad durchlaufen:

$$
\begin{array}{rlllll}
\pi = & (\mathtt{K}, & \{\mathtt{n} \mapsto x_1, & \mathtt{s} \mapsto x_2, & \mathtt{p} \mapsto x_3, & \mathtt{i} \mapsto x_4\}) \quad \mathtt{n = read();} \\
& (\mathtt{J}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto x_2, & \mathtt{p} \mapsto x_3, & \mathtt{i} \mapsto x_4\}) \quad \mathtt{s = 3;} \\
& (\mathtt{I}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 3, & \mathtt{p} \mapsto x_3, & \mathtt{i} \mapsto x_4\}) \quad \mathtt{p = 1;} \\
& (\mathtt{H}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 3, & \mathtt{p} \mapsto 1, & \mathtt{i} \mapsto x_4\}) \quad \mathtt{i = 0;} \\
& (\mathtt{A}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 3, & \mathtt{p} \mapsto 1, & \mathtt{i} \mapsto 0\}) \quad \mathtt{i! = n - 1} \\
& (\mathtt{D}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 3, & \mathtt{p} \mapsto 1, & \mathtt{i} \mapsto 0\}) \quad \mathtt{p = 3 * p;} \\
& (\mathtt{C}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 3, & \mathtt{p} \mapsto 3, & \mathtt{i} \mapsto 0\}) \quad \mathtt{s = s + 2 * p;} \\
& (\mathtt{B}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 9, & \mathtt{p} \mapsto 3, & \mathtt{i} \mapsto 0\}) \quad \mathtt{i = i + 1;} \\
& (\mathtt{A}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 9, & \mathtt{p} \mapsto 3, & \mathtt{i} \mapsto 1\}) \quad \mathtt{i! = n - 1} \\
& (\mathtt{D}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 9, & \mathtt{p} \mapsto 3, & \mathtt{i} \mapsto 1\}) \quad \mathtt{p = 3 * p;} \\
& (\mathtt{C}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 9, & \mathtt{p} \mapsto 9, & \mathtt{i} \mapsto 1\}) \quad \mathtt{s = s + 2 * p;} \\
& (\mathtt{B}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 27, & \mathtt{p} \mapsto 9, & \mathtt{i} \mapsto 1\}) \quad \mathtt{i = i + 1;} \\
& (\mathtt{A}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 27, & \mathtt{p} \mapsto 9, & \mathtt{i} \mapsto 2\}) \quad \mathtt{!(i! = n - 1)} \\
& (\mathtt{F}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 27, & \mathtt{p} \mapsto 9, & \mathtt{i} \mapsto 2\}) \quad \mathtt{write(s);} \\
& (\mathtt{E}, & \{\mathtt{n} \mapsto 3, & \mathtt{s} \mapsto 27, & \mathtt{p} \mapsto 9, & \mathtt{i} \mapsto 2\})
\end{array}
$$

b) Wir raten als erstes die Schleifen-Invariante A. Dabei versuchen wir s und p mit i auszudrücken.

$$\mathtt{A} :\equiv \mathtt{p} = 3^{\mathtt{i}} \wedge \mathtt{s} = 3^{\mathtt{i}+1}$$

Es folgt:

$$
\begin{array}{rl}
\mathbf{WP}[\![\mathtt{i = i + 1;}]\!](\mathtt{A}) & \equiv \mathtt{p} = 3^{\mathtt{i}+1} \wedge \mathtt{s} = 3^{\mathtt{i}+2} \equiv: \mathtt{B} \\
\mathbf{WP}[\![\mathtt{s = s + 2 * p;}]\!](\mathtt{B}) & \equiv \mathtt{p} = 3^{\mathtt{i}+1} \wedge \mathtt{s} + 2 \cdot \mathtt{p} = 3^{\mathtt{i}+2} \equiv: \mathtt{C} \\
\mathbf{WP}[\![\mathtt{p = 3 * p;}]\!](\mathtt{C}) & \equiv 3 \cdot \mathtt{p} = 3^{\mathtt{i}+1} \wedge \mathtt{s} + 2 \cdot 3 \cdot \mathtt{p} = 3^{\mathtt{i}+2} \\
& \Leftarrow \mathtt{p} = 3^{\mathtt{i}} \wedge \mathtt{s} = 3^{\mathtt{i}+2} - 2 \cdot 3^{\mathtt{i}+1} = 3 \cdot 3^{\mathtt{i}+1} - 2 \cdot 3^{\mathtt{i}+1} = 3^{\mathtt{i}+1} \\
& \equiv \mathtt{A} \equiv: \mathtt{D} \\
\mathbf{WP}[\![\mathtt{write(s);}]\!](\mathtt{E}) & \equiv \mathtt{E} \equiv: \mathtt{F} \\
\mathbf{WP}[\![\mathtt{i! = n - 1}]\!](\mathtt{F}, \mathtt{A}) & \equiv (\mathtt{i} = \mathtt{n} - 1 \wedge \mathtt{F}) \vee (\mathtt{i} \neq \mathtt{n} - 1 \wedge \mathtt{A}) \\
& \equiv (\mathtt{i} = \mathtt{n} - 1 \wedge \mathtt{s} = 3^{\mathtt{n}}) \vee (\mathtt{i} \neq \mathtt{n} - 1 \wedge \mathtt{A}) \\
& \Leftarrow (\mathtt{i} = \mathtt{n} - 1 \wedge \mathtt{s} = 3^{\mathtt{i}+1} \wedge \mathtt{p} = 3^{\mathtt{i}}) \vee (\mathtt{i} \neq \mathtt{n} - 1 \wedge \mathtt{A}) \\
& \equiv (\mathtt{i} = \mathtt{n} - 1 \wedge \mathtt{A}) \vee (\mathtt{i} \neq \mathtt{n} - 1 \wedge \mathtt{A}) \\
& \equiv (\mathtt{i} = \mathtt{n} - 1 \vee \mathtt{i} \neq \mathtt{n} - 1) \wedge \mathtt{A} \\
& \equiv \mathbf{true} \wedge \mathtt{A} \\
& \equiv \mathtt{A} \\
\mathbf{WP}[\![\mathtt{i = 0;}]\!](\mathtt{A}) & \equiv \mathtt{p} = 3^0 = 1 \wedge \mathtt{s} = 3^1 = 3 \equiv: \mathtt{H} \\
\mathbf{WP}[\![\mathtt{p = 1;}]\!](\mathtt{H}) & \equiv \mathtt{s} = 3^1 = 3 \equiv: \mathtt{I} \\
\mathbf{WP}[\![\mathtt{s = 3;}]\!](\mathtt{I}) & \equiv \mathbf{true} \equiv: \mathtt{J} \\
\mathbf{WP}[\![\mathtt{n = read();}]\!](\mathtt{J}) & \equiv \forall \mathtt{n}.\mathbf{true} \equiv \mathbf{true} \equiv: \mathtt{K}
\end{array}
$$