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- ... and even any **partial information** about x should not be leaked.

→ This process is insecure: $\text{send}_c\langle \{0\}_x \rangle; \text{halt}$

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- But one can get **partial information** about x .

For example, one can find out whether x is 0 or not, by using the property:

$$x = 0 \quad \text{iff} \quad \{0\}_x = \{0\}_0$$

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For example, one can find out whether x is 0 or not, by using the property:

$$x = 0 \quad \text{iff} \quad \{0\}_x = \{0\}_0$$

→ $P(x)$ does not preserve the secrecy of x .

$$P(x) \triangleq \text{send}_c \langle \{0\}_x \rangle; \text{halt}$$

In spi-calculus terminology, we consider the test (Q, \bar{d}) , where

$$Q \triangleq \text{recv}_c(y); Q_1(y) \quad Q_1(y) \triangleq \text{check } (y == \{0\}_0); Q_2 \quad Q_2 \triangleq \text{send}_d \langle 0 \rangle; \text{halt}$$

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$$P(0) \xrightarrow{\bar{c}} \langle \{0\}_0 \rangle \text{halt} \quad \text{and} \quad Q \xrightarrow{c} (y)Q_1(y)$$

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$$P(0) \mid Q \xrightarrow{\tau} \bar{c} \langle \{0\}_0 \rangle \text{halt} \quad @ \quad (y)Q_1(y)$$

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$$\begin{aligned} P(0) \mid Q &\xrightarrow{\tau} \bar{c} \langle \{0\}_0 \rangle \text{halt} \quad @ \quad (y)Q_1(y) \\ &= \text{halt} \mid Q_1(\{0\}_0) \end{aligned}$$

$$P(x) \triangleq \text{send}_c \langle \{0\}_x \rangle; \text{halt}$$

$$Q \triangleq \text{recv}_c(y); Q_1(y) \quad Q_1(y) \triangleq \text{check } (y == \{0\}_0); Q_2 \quad Q_2 \triangleq \text{send}_d \langle 0 \rangle; \text{halt}$$

$$Q_1(\{0\}_0) > Q_2 \xrightarrow{\bar{d}} \langle 0 \rangle \text{halt}$$

$$\text{Hence } Q_1(\{0\}_0) \xrightarrow{\bar{d}} \langle 0 \rangle \text{halt}$$

$$\begin{aligned} \text{Hence } \text{halt} \mid Q_1(\{0\}_0) &\xrightarrow{\bar{d}} \text{halt} \mid \langle 0 \rangle \text{halt} \\ &= \langle 0 \rangle (\text{halt} \mid \text{halt}) \end{aligned}$$

Hence $P(0)$ passes the test (Q, \bar{d}) .

And we can check that $P(\text{succ } (0))$ does not pass the test (Q, \bar{d}) .

Similarly the following challenge response step **does not preserve secrecy** of K_{ab} in the spi-calculus model, although the key K_{ab} cannot be computed by an attacker.

$$A \longrightarrow B : N_a$$

$$B \longrightarrow A : \{N_a\}_{K_{ab}}$$

One session of the protocol can be represented by the process

$$\text{new } K; (\text{new } N; \text{send}_c\langle N \rangle; \text{halt} \mid \text{recv}_c(x); \text{send}_c\langle \{x\}_K \rangle; \text{halt})$$

Intuitively, an attacker can send a desired message in place of N_a and then get partial information about the secret key as in the previous example.

Another example: $P_1(x) \triangleq \text{new } K; \text{send}_c\langle\{x\}_K\rangle; \text{halt}.$

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Consider arbitrary terms M_1 and M_2 . We show that:

if $P_1(M_1)$ passes some test (Q, β) then $P_1(M_2)$ also passes the test (Q, β)

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For this we show that for all $n \geq 0$ and for all R , if $P_1(M_1) \mid R$ can make a sequence of actions β_1, \dots, β_n then $P_1(M_2) \mid R$ can also do so.

Note: we must have $\beta_i = \tau$ for $i \leq n - 1$.

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By induction on n .

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Note: we must have $\beta_i = \tau$ for $i \leq n - 1$.

By induction on n . For $n = 0$ there is nothing to prove.

$$P_1(x) \triangleq \text{new } K; \text{send}_c \langle \{x\}_K \rangle; \text{halt}$$

Case 1: the left component makes an action.

$$P_1(M_1) \mid R \xrightarrow{\bar{c}} (\text{new } K) \langle \{M_1\}_K \rangle (\text{halt} \mid R)$$

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Then we also have

$$P_1(M_2) \mid R \xrightarrow{\bar{c}} (\text{new } K)\langle\{M_2\}_K\rangle(\text{halt} \mid R)$$

No further transitions are possible in either case. Hence we are done.

Case 2: the right component R is a process and makes an action.

$R \xrightarrow{\beta_1} B$ so that $P_1(M_1) \mid R \xrightarrow{\beta_1} P_1(M_1) \mid B$

and $P_1(M_1) \mid B$ makes a sequence of actions β_2, \dots, β_n .

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Then we also have $P_1(M_2) \mid R \xrightarrow{\beta_1} P_1(M_2) \mid B$

and by **induction hypothesis**, $P_1(M_2) \mid B$ makes a sequence of actions β_2, \dots, β_n .

$$P_1(x) \triangleq \text{new } K; \text{send}_c \langle \{x\}_K \rangle; \text{halt}$$

Case 3: the two components communicate over channel c .

$$P_1(M_1) \xrightarrow{\bar{c}} (\text{new } K) \langle \{M_1\}_K \rangle \text{halt}$$

and $R \xrightarrow{c} (y)R'$

so that $P_1(M_1) \mid R \xrightarrow{\tau} \text{new } K; (\text{halt} \mid R'(\{M_1\}_K))$

and $\text{new } K; (\text{halt} \mid R'(\{M_1\}_K))$ makes the sequence of actions β_2, \dots, β_n .

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$$P_1(x) \triangleq \text{new } K; \text{send}_c \langle \{x\}_K \rangle; \text{halt}$$

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It remains to show that ...

Claim: For all $S(x)$, if $K \notin fn(S)$, and if $S(\{M_1\}_K) \xrightarrow{\beta} A$ then

- $A = B(\{M_1\}_K)$ with $K \notin fn(S)$
- $S(\{M_2\}_K) \xrightarrow{\beta} B(\{M_1\}_K)$

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Proof: by structural induction on S .

Case 1: $S = \text{send}_c \langle N_1, \dots, N_k \rangle; S_1$

We have $S(\{M_1\}_K) \xrightarrow{\bar{c}} (\langle N_1, \dots, N_k \rangle S_1)(\{M_1\}_K)$

and also $S(\{M_2\}_K) \xrightarrow{\bar{c}} (\langle N_1, \dots, N_k \rangle S_1)(\{M_2\}_K)$

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Case 2: $S = \text{recv}_c(x_1, \dots, x_k); S_1$

We have $S(\{M_1\}_K) \xrightarrow{c} (x_1, \dots, x_k) S_1(\{M_1\}_K)$

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Case 3: $S = \text{halt}$. Trivial, since no actions are possible.

Case 4: $S = S_1 \mid S_2$,

and $S_1(\{M_1\}_K) \xrightarrow{\beta} A_1$

so that $S(\{M_1\}_K) \xrightarrow{\beta} A_1 \mid S_2(\{M_1\}_K)$

By induction hypothesis, $A_1 = B_1(\{M_1\}_K)$, $K \notin \text{fn}(B_1)$ and $S_1(\{M_2\}_K) \xrightarrow{\beta} B_1(\{M_2\}_K)$.

Then we have $S(\{M_2\}_K) \xrightarrow{\beta} B_1(\{M_2\}_K/x) \mid S_2(\{M_2\}_K)$.

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By induction hypothesis, $A_1 = B_1(\{M_1\}_K)$, $K \notin \text{fn}(B_1)$ and $S_1(\{M_2\}_K) \xrightarrow{\beta} B_1(\{M_2\}_K)$.

Then we have $S(\{M_2\}_K) \xrightarrow{\beta} B_1(\{M_2\}_K/x) \mid S_2(\{M_2\}_K)$.

We argue similarly if the right component S_2 makes an action.

Case 5: $S = S_1 \mid S_2$,

$$S_1(\{M_1\}_K) \xrightarrow{\bar{c}} A_1 \quad \text{and} \quad S_2(\{M_1\}_K) \xrightarrow{c} A_2$$

so that $S(\{M_1\}_K) \xrightarrow{\tau} A_1 @ A_2$

By induction hypothesis,

$$A_1 = B_1(\{M_1\}_K), \quad A_2 = B_2(\{M_1\}_K), \quad K \notin \text{fn}(B_1) \cup \text{fn}(B_2),$$

$$S_1(\{M_2\}_K) \xrightarrow{\bar{c}} B_1(\{M_2\}_K) \quad \text{and} \quad S_2(\{M_2\}_K) \xrightarrow{c} B_2(\{M_2\}_K).$$

Hence $S(\{M_2\}_K) \xrightarrow{\tau} B_1(\{M_2\}_K) @ B_2(\{M_2\}_K)$

Case 6: $S = \text{repeat } S_1$ and

- either $S_1(\{M_1\}_K) \xrightarrow{\beta} A$ so that $S(\{M_1\}_K) \xrightarrow{\beta} A \mid S(\{M_1\}_K)$.
- or $S_1(\{M_1\}_K) \xrightarrow{\bar{c}} A_1$ and $S_2(\{M_1\}_K) \xrightarrow{c} A_2$
so that $S(\{M_1\}_K) \xrightarrow{\tau} (A_1 @ A_2) \mid S(\{M_1\}_K)$

The cases are similar to Case 4 and Case 5.

Case 6: $S = \text{repeat } S_1$ and

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The cases are similar to Case 4 and Case 5.

Case 7: $S = \text{new } n; S_1$. Again a straightforward application of induction hypothesis.

Case 8: $S = \text{check } (M == N); S_1$,

$M[\{M_1\}_K/x] = N[\{M_1\}_K/x]$ and $S_1(\{M_1\}_K) \xrightarrow{\beta} A$ so that $S(\{M_1\}_K) \xrightarrow{\beta} A$

Since $K \notin \text{fn}(M) \cup \text{fn}(N)$, we have $M = N$. (Proof: exercise)

Hence $M[\{M_2\}_K/x] = N[\{M_2\}_K/x]$.

Also by induction hypothesis, $A = B(\{M_1\}_K)$ and

$S_1(\{M_2\}_K) \xrightarrow{\beta} B(\{M_2\}_K)$

so that $S(\{M_2\}_K) \xrightarrow{\beta} B(\{M_2\}_K)$.

Case 9: $S = \text{let } (x, y) = M; S_1$

Case 10: $S = \text{case } M \text{ of } 0 : S_1, \text{succ } (y) : S_2$

These are similar to (and simpler than) Case 11.

Case 11: $S = \text{case } M \text{ of } \{x_1, \dots, x_k\}_N : S_1$.

$K \neq N$ because $K \notin \text{fn}(S)$.

Hence if M is the variable x then no action is possible.

For an action to be possible we must have $M = \{N_1, \dots, N_k\}_N$.

Let $S_1[N_1/x_1, \dots, N_k/x_k][\{M_1\}_K/x] \xrightarrow{\beta} A$

so that $S[\{M_1\}_K/x] \xrightarrow{\beta} A$.

By induction hypothesis, $A = B[\{M_1\}_K/x]$ and

$S_1[N_1/x_1, \dots, N_k/x_k][\{M_2\}_K/x] \xrightarrow{\beta} B[\{M_2\}_K/x]$

so that $S[\{M_2\}_K/x] \xrightarrow{\beta} B[\{M_2\}_K/x]$.

After all this, we conclude:

the process $\text{new } K; \text{send}_c\langle\{x\}_K\rangle; \text{halt}$ preserves the secrecy of x .

Unfortunately too **tedious proof** for an extremely simple protocol.

Need simpler methods of showing security of a protocol....

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We define rules for controlling the **flow of information** in the protocol.

Information flow analysis for the Spi-calculus

- Information flow analysis is used in various programming languages (imperative, functional, object-oriented languages, process calculi,...) to study security properties.
- Data is classified into various **security levels** representing varying degrees of confidentiality.
- A program is secure if information from more confidential data does not flow to less confidential data.

Consider the C language.

Assume variable x has security level high and variable y has security level low.

Then the following statement cannot be allowed in the program:

$$y^{low} = x^{high} + 1;$$

By reading the less confidential data y , we can get information about the high confidential data x .

The following statement is fine.

$$x^{high} = y^{low} + 1;$$

The following code should be disallowed.

```
z = 2 * xhigh + 1;  
if (z > 100)  
    ylow = 10;  
else  
    ylow = 20;
```

By observing **y** we can get **some** information about **x**.

→ **Implicit flows** should also be controlled.

For the Spi-calculus ...

We classify data into three classes

secret data which should not be leaked

public data which can be communicated to anyone

any arbitrary data

Subsumption relation on classes:

secret \preceq **any**

public \preceq **any**

T \preceq T for $T \in \{\mathbf{secret}, \mathbf{public}, \mathbf{any}\}$

Some initial ideas.

- Secret data should not be sent on public channels.
- Secret data should encrypted with public key should not be public.
- Public data encrypted with secret key may be made public.
- Data encrypted with private key may be made public.

We formulate these as a set of **typing rules**.

type of message M = secrecy level of M

process P is well-typed = P does not allow bad information flow